Low-velocity impact cratering experiments in granular slopes

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A R T I C L E   I N F O

Article history:
Received 31 July 2016
Revised 21 March 2017
Accepted 22 March 2017
Available online 23 March 2017

Keywords:
Granular slopes
Impact processes
Asymmetric craters
Scaling
Asteroid Vesta

A B S T R A C T

Low-velocity impact cratering experiments are conducted in sloped granular targets to study the effect of the slope angle $\theta$ on the crater shape and its scales. We use two types of granular matter, sand and glass beads, formed of which has a larger friction coefficient $\mu_s = \tan \theta_r$, where $\theta_r$ is the angle of repose. Experiments show that as $\theta$ increases, the crater becomes shallower and elongated in the direction of the slope. Furthermore the crater floor steepens in the upslope side and a thick rim forms in the downslope side, thus forming an asymmetric profile. High-speed images show that these features are results of ejecta being dispersed farther towards the downslope side and the subsequent avalanche which buries much of the crater floor. Such asymmetric ejecta dispersal can be explained by combining the Z-model and a ballistic model. Using the topographic maps of the craters, we classify crater shape regimes I-III, which transition with increasing $\theta$ : a full-rim crater (I), a broken-rim crater (II), and a depression (III). The critical $\theta$ for the regime transitions are larger for sand compared to glass beads, but collapse to close values when we use a normalized slope $\tilde{\theta} = \tan \theta / \tan \theta_r$. Similarly we derive $\tilde{\theta}$-dependences of the scaled crater depth, length, width and their ratios which collapse the results for different targets and impact energies. We compare the crater profiles formed in our experiments with deep craters on asteroid Vesta and find that some of the scaled profiles nearly overlap and many have similar depth / length ratios. This suggests that these Vestan craters may also have formed in the gravity regime and that the formation process can be approximated by a granular flow with a similar effective friction coefficient.

1. Introduction

Recent orbital mission observations of the asteroid Vesta by the spacecraft Dawn, have revealed the existence of many asymmetric impact craters on its slopes (Jaumann et al., 2012). Such craters are characterized by a wider upslope wall with a sharp crest and a narrower downslope wall with a smoothed rim indicating that it is covered by an avalanched material. A detailed investigation (Krohn et al., 2014a) has shown that asymmetric craters on Vesta have a wide range of morphologies, and numerical simulations have demonstrated that such craters indeed form by impact on slopes. Impact craters on slopes have also been discovered on asteroid Lutetia (Elbeshanuen et al., 2011) and on the Earth’s moon (Plescia, 2012). It appears that these craters are common on asteroids and satellites which have a large relief to size ratios.

In addition to numerical simulations, impact cratering experiments on slopes provide information which help to understand how such craters form and how their characteristic scales depend on the parameters such as the physical properties of the target, slope angle and impact energy. However most of the impact experiments on slopes have been conducted from a different perspective, to understand the effect of rain splash on slopes (e.g., Furbish et al., 2007), a process considered to cause downslope mass movement (Melosh, 2011). Experiments on sloped targets using solid impactors are few (Aschauer et al., 2016), and needs further investigation.

Inspired by the discoveries on Vesta, we conducted a series of impact cratering experiments, in which a vertically free-falling sphere impacts into a sloped granular target. We vary the target granular matter, target slope and the impact energy, to study how the crater shape changes with these parameters. High-speed images are used to study the process of crater formation. Topographic maps of the craters are constructed, which are used to classify the shape regimes and to study how the characteristic scales of the craters depend on these parameters. Profiles of the craters obtained from these maps are compared with those on Vesta to infer the conditions under which they formed.
2. Experimental methods

Fig. 1 shows the experimental setup which is modified from that used in Takita and Sumita (2013). We use an acrylic rectangular container with a width 180 mm, length 250 mm and a depth of 90 mm, which is filled with granular matter (Fig. 2(a),(b)). The details of the granular matter used are described in the next section. For the impactor we use a stainless steel (SUS440C) sphere (sphericity 0.7 μm) with a density of ρi = 7700 kg/m3. We use spheres with 3 different diameters (D) and masses (m), which are listed in Table 1. The sphere is dropped from a height h (h = 1013 – 1461 mm) using an electromagnet. The container width is > 8D, and is wide enough so that the effects of the container walls on the impactor can be considered to be negligibly small (Seguin et al., 2008). The container depth is deep enough so that the impactor stops before it reaches the bottom.

We conduct experiments under 5 impact conditions with different combinations of D and h so that the resulting impact velocity v_i = √(2gh) and impact energies E = mgh = πD^3ρi gh/6 are different, which are summarized in Table 1. Since E differs among the 5 conditions, we hereafter use E to represent the impact condition. Our experiments cover E range of 1 order of magnitude.

The topography of the granular surface is measured using a 2-D laser profilometer (Omron, ZG2-WDS70) which measures the topography in the width band of ~70 mm. The resolution of the profilometer is ~0.1 mm in the horizontal direction and ~0.006 mm in the vertical direction. The profilometer is attached to a stepping motor (COMS, PM8-B-200X) which moves in the downslope direction and the measurements are made every 0.2 mm. This resolution is comparable to the particle diameter d of the granular matter used in our experiments (Table 2). The tilt of the stepping motor relative to the horizontal is measured using a digital tiltmeter. The x, y and z coordinates are indicated in Fig. 1. Here the y axis is in the strike direction of the target surface and the x–y plane defines the horizontal plane.

The experimental procedure is as follows. We fill the container with granular matter and form a flat surface using a ruler. For each experiment we use the same filling procedure so that the volumetric packing fraction φ of the granular matter (Table 2) varies little among the different experimental runs. We then position the container on a hinged wooden board and insert a spacer to a specified position, and tilt the board to an angle θ. We conduct a total of 3 profilometry scans to cover the whole width range of the container before and after the impact cratering. We construct a topography grid data with a spatial resolution of 0.2 mm in the x–y plane using MATLAB. We then construct a displacement (∆z) map by subtracting the map of the original surface from the map after cratering. After each impact cratering experiment, a crater is illuminated by a halogen lamp from the side and a photo is taken by

![Fig. 1. An experimental setup. h is the drop height, θ is the slope angle. The y axis is in the strike direction of the target surface. A 2-D laser profilometer is attached to a stepping motor and scans the target surface in the downslope direction (red broken arrow). 3 scans of the target surface are made by changing the position of the profilometer in the y direction (blue broken arrow).](image)

![Fig. 2. (a) Sand and (b) glass beads used for the target granular matter in our experiments. (c) Shear rheology measurements of sand (red line) and glass beads (blue line). A vane spindle (see inset figure) is inserted vertically into the granular matter to a depth of Zvane = 11.76 mm, and rotates at 300 rpm. Shear stress data are 5 point running averaged.](image)

<table>
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<th>D (mm)</th>
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<th>h (mm)</th>
<th>v_i (m/s)</th>
<th>E (J)</th>
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Table 1
A summary of the impact conditions. D: impactor diameter, m: impactor mass, h: drop height, v_i: impact velocity, E: impact energy.
a digital camera (Casio EX-ZR200). We conducted a total of 42 and 33 experimental runs using sand and glass beads targets respectively, and constructed the topographic maps of the craters. We also conducted additional experiments to record the whole process of crater formation from different viewing points using a high-speed camera (IDT Motion Scope M3) at 1000 frame/s. Images obtained from the 22 side view experiments were used to measure the ejection angles and velocities towards the upslope and downslope directions.

3. Physical properties of granular matter

We use two types of well-sorted granular matter; angular beach sand (Chirihama, Japan: Fig. 2(a)) and spherical soda-lime glass beads (Toshin Roch, Japan: Fig. 2(b)). The particle diameters (d) of the granular matter are d ~ 0.2 mm (Table 2) and are larger than the critical size of d ~ 0.1 mm above which the cohesion from moisture becomes negligible (Duran, 2000; Andreotti et al., 2013). Sand is the same as that used in Takita and Sumita (2013).

The packing fractions (φ) of the granular matter are chosen so that a stable slope forms at the steepest slope angle used in the experiments. This is the reason for a larger φ = 0.57 for the experiments using sand target compared to φ = 0.52 used in Takita and Sumita (2013). φ of the glass beads is between the random loose packing of spheres (φ = 0.55) (Andreotti et al., 2013) and random close packing of spheres (φ = 0.64) (Mavko et al., 1998).

Frictional properties of granular matter are characterized by the static and dynamic friction coefficients μs and μd. μs is related to the angle of repose θr, as

\[ μ_s = \tan θ_r, \]

We measured θr in a 2-D concave geometry (Duran, 2000) as follows. An acrylic container with a horizontal cross section of 62 mm × 44 mm and a height of 95 mm, is filled with granular matter. The granular matter is then allowed to flow out from a 2 mm wide slit at the bottom center of the container, and we measure the slope angle of the resulting granular slope. Similarly μd is obtained from

\[ μ_d = \tan θ_c, \]

where θc is the collapse angle, which is the slope angle of the crater floor formed in a horizontal target (see Fig. 3). θc is calculated using the crater diameter and depth, as expressed in Eq. (8). We use this angle because it corresponds to the slope angle at which the granular matter stops flowing. The value of θc given in Table 2 is calculated from the average of the crater floor slopes formed at different impact energies. It is known that a steady uniform dense granular flow stops below a critical slope angle (Pouliquen, 1999), and we consider that θc is related to this limit. Table 2 shows that μs, μd of sand are larger than those of glass beads, which is consistent with the previous measurements (Duran, 2000). The ratio of these friction coefficients (μd/μs) are μd/μs ≈ 0.58 for sand and μd/μs ≈ 0.43 for glass beads, both of which are around 0.5.

Coulomb’s friction law relates the friction coefficient μ to the yield stress (strength) σy of the granular target

\[ σ_y = μP, \]

where P is the normal stress. In the next section, we compare σy with other stresses to identify the dominant stresses during impact cratering.

We use two methods to estimate σy. First is to use μ = μγ. Since the depth scale of cratering is comparable to the size of the impactor (~ D) (Takita and Sumita, 2013), which is also verified in our experiments, we estimate P = μγ P where and obtain σy = 128 – 258 and 85 – 171 Pa for sand and glass beads targets, respectively. Here a factor of ~ 2 range arises from different D used in the experiments (Table 1).

Second is to use a rheometer to measure σy (Takita and Sumita, 2013). The method is as follows. First we fill a beaker (radius 21 mm) with a specified mass of granular matter. The beaker is then tapped so that the granular matter compacts until its bulk volume is 50 cm3 and φ becomes the same as those of the target (Table 2). Next we insert a 4-bladed vane spindle (Brookfield V74: radius 2945 mm, height 117.6 mm, see inset of Fig. 2(c)), vertically into the sample so that its upper end is just beneath the surface of the sample. Then we shear the sample using a rheometer (Physica MCR 301, Anton Paar), at 300 rotation per minute (rpm) which corresponds to a shearing velocity of 0.09 m/s at the radius of the vane spindle. Torque M is measured at a sampling frequency of 10 Hz, from which we calculate the shear stress σ (Supplementary Material). Results of the measurements are shown in Fig. 2(c). Here the spindle starts to rotate at t = 0 s and adjusts to 300 rpm in about 0.1 s. From the steady-state values we obtain σy = 268 Pa for sand and σy = 82 Pa for glass beads.

We further calculate μ from these measurements (Supplementary Material) and obtain μ = 2.90 and 0.87 for sand and glass beads respectively. Although these values are 3.6 and 1.7 times larger than their respective μs, a larger μ for sand compared to glass beads is common with μs and μd. We conducted measurements by inserting the spindle to different average depths z and find that μ increases with z (see Supplementary Fig. 1). The thickness of the granular flow during the formation of a granular slope which is used to measure the angle of repose, is limited to < 10d (Duran, 2000). It seems that μ increases with the depth scale of the granular flow. We therefore consider that μs is a lower bound estimate of μ.

4. Normalized target slope and dimensionless numbers

The main parameter in our experiments is the target slope angle θ which is varied in the range of 0 ≤ θ < θr. Since sand has a larger θr compared to glass beads, we use sand to study the θ-dependence in detail, and then use glass beads to study how the different granular friction affects the crater shape and the θ-dependence.

We normalize θ using θr as

\[ \hat{θ} = \left( \frac{\tan θ}{\tan θ_r} \right), \]

and define $\hat{θ}$ as the normalized target slope. It follows that the experiments are conducted at 0 ≤ $\hat{θ}$ < 1. We use $\hat{θ}$ to normalize the results of sand and glass beads which have different μs(= tan θr).
Relevant stresses during impact cratering can be evaluated using the following two dimensionless numbers (Takita and Sumita, 2013; Katsuragi, 2016). First is the dimensionless gravity

$$ g' = \frac{\phi \rho g H}{\sigma_y} = \frac{\phi \pi_2 \pi_4}{\pi_3}, \tag{5} $$

which compares the granular pressure at a depth of $H$ to the target yield stress $\sigma_y$. For $g' \gg 1$ the excavated transient crater collapses by gravity (a gravity regime) whereas for $g' \ll 1$ the target strength withstands against collapse (a strength regime). In this study we use $g'$ because it best distinguishes the two end member cases of impact cratering. Second is the dimensionless inertia

$$ I' = \frac{\rho_i \nu^2}{\sigma_y} = \frac{1}{\pi_3}, \tag{6} $$

which compares the inertial stress to the yield stress. For $I' \gg 1$ target excavation is possible and vice versa for $I' \ll 1$. $g'$ and $I'$ can also be expressed using the commonly used following dimensionless parameters (Holsapple, 1993). $\pi_2 = g H / \nu^2$ (an inverse Froude number), $\pi_3 = \sigma_y / \rho_i \nu^2$ (an inverse Cauchy number) and $\pi_4 = \rho_i / \rho_t$, as shown in Eqs. (5), (6).

Here we estimate $g'$ and $I'$ in our experiments. $\sigma_y$ of a granular target can be estimated from Eq. (3) as

$$ \sigma_y = \mu \phi \rho g H, \tag{7} $$

where $H$ is the depth of scale cratering. Using a lower bound estimate of $\mu = \mu_5$ (Section 3) and $H \sim D$, we obtain upper bound estimates of $g' \approx 13 \cdot 790 \leq I' \leq 1590$ for sand target experiments and $g' \approx 19 \cdot 1190 \leq I' \leq 2390$ for glass beads target experiments. Our rheology measurements indicate that $\mu$ increases with the depth scale of granular flow (Supplementary Fig. 1). For the depth scale $\sim D$ of cratering we estimate $\mu \sim 3.1 - 5.6 \mu_4$ (Supplementary Material). Although the estimate for $\mu$ has an uncertainty it is sufficient to provide an order of magnitude estimates for $g'$ and $I'$, which become $g' = O(0.1 - 1)$ and $I' = O(10^{-2} - 10^3)$ for both the sand and glass beads target experiments. This estimate for $g'$ indicate that our experiments are in the gravity regime or in the transition of gravity and strength regimes, which is consistent with the observation that the crater wall collapses in all of our experiments. The estimate $I' \gg 1$ is consistent with the observation that crater excavation is possible. Impact cratering and subsequent collapse occurs despite a low impact velocity because we use non-cohesive granular matter for the target (see Katsuragi, 2016 for a review).

We also evaluate the ratio $I'/g' = \rho_i \nu^2 / (\phi \rho g D)$, which is equivalent to the ratio of the impact energy to the work needed to lift the excavated mass upwards by a displacement $D$, within an order of magnitude. We obtain $I'/g' \approx 630 - 1260$ and $I'/g' \approx 610 - 1240$ for experiments using sand and glass beads targets, respectively. These large values ($I'/g' \gg 1$) indicate that only a small fraction ($\sim O(10^{-3})$) of the impact energy is used for the work needed to lift the excavated mass upwards, which has been remarked by de Vet and de Bruyn (2007).

5. Results

5.1. Craters formed in horizontal target

Fig. 3 compares crater profiles formed in horizontal ($\theta = 0^\circ$), sand and glass beads targets, at an impact energy of $E = 0.58$ J. These craters have a bowl-like shape and closely resemble the classic simple craters (Melosh, 2011). Comparing the profiles, we find that the crater formed in glass sand is shallower and wider than the crater formed in sand. Furthermore the profile of a crater in glass beads is curved with a central peak at the bottom, whereas the profile of a crater in sand is straight without a central peak. It is well known that a central peak forms when the wall of an excavated transient crater collapses and a granular jet ascends from the center of the crater floor (e.g., Walsh et al., 2003). We confirmed from high-speed images that a jet ascent indeed occurs in our experiments using glass beads target (see Movie 4 for a case of a sloped target).

We define the depth and diameter of a crater as shown in Fig. 3 and use these to calculate the collapse angle $\theta_c$. Here the depth is defined relative to the original surface and is the largest negative displacement. Diameter is also defined at the surface. Using these scales we calculate $\theta_c$ from

$$ \theta_c = \arctan \left( \frac{\text{Depth}}{\text{Diameter}/2} \right). \tag{8} $$

We note that this definition for $\theta_c$ does not consider the profile curvature and therefore is a mean crater floor slope angle. For the craters formed in sand and glass beads targets shown in Fig. 3, the crater floor slopes are $\tan \theta_c = 0.454$ and 0.201, which correspond to $\theta_c = 24.4^\circ$ and $11.4^\circ$, respectively.

5.2. Craters formed in sloped targets

Fig. 4 shows how the shapes of the impact craters formed in sand targets change with increasing slope angle $\theta$ at an impact energy of $E = 0.58$ J. This figure shows a striking change of the crater shape with $\theta$. As $\theta$ increases, the crater becomes shallower and elongated in the direction of the slope. The rim becomes thicker in the downslope side and thinner in the upslope side such that it forms a crescent shape, which is already evident at $\theta = 11^\circ$. At $\theta = 34^\circ$ close to the angle of repose of sand, the crater is mostly buried by the overall avalanche towards the downslope direction which follows after the impact. We also note that terraces form in the crater floor as a result of avalanche. We locate the impact points within the crater floor using high-speed images taken from a direction perpendicular to the slope plane, examples of which are shown in the Supplementary Fig. 2.

Fig. 5(a) shows the topographic maps of the 4 craters selected from Fig. 4. From the topographic maps of the craters, we subtract the original surfaces before cratering and obtain the displacement $\delta z$ maps, which are shown in Fig. 5(c). Displacement maps highlight the change of topography from cratering. The origins of these maps are defined at the points of largest negative $\delta z$, and black lines in Fig. 5(a) indicate $y = 0$. The region which is lower than the original surface $\delta z < 0$ is defined as the crater floor. Fig. 5(b) shows the profiles at $y = 0$. These profiles show that as the target
Fig. 4. Impact craters formed in sand targets at different slope angles $\theta$ at an impact energy of $E = 0.58$ J. Photos are taken from a direction perpendicular to the surface. Downslope direction is towards the left and the light source is on the right. Scale bars indicate 10 mm. $\theta = 0 - 16^\circ$ cases are full-rim craters (regime I), $\theta = 22 - 26^\circ$ cases are broken-rim craters (regime II), and $\theta = 34^\circ$ case is a depression (regime III).

(a) 
- $\theta = 0^\circ$: Full rim crater
- $\theta = 11^\circ$: Full rim crater
- $\theta = 22^\circ$: Broken rim crater
- $\theta = 34^\circ$: Depression

(b) 
- Topographies of the 4 craters selected from those shown in Fig. 4. The $y$-axis is in the strike direction (Fig. 1) and the origin $(x, y) = (0, 0)$ is defined at the point of largest negative displacement ($\delta z$) relative to the original surface. A black line indicates $y = 0$. (b) Crater profiles (red lines) at $y = 0$. Black broken lines indicate the origin surface. $\cdot$ indicate the largest negative $\delta z$ which defines the origin $(x = 0)$. (c) Displacement ($\delta z$) maps. Contours are drawn at 2 mm intervals. Black broken contours indicate $\delta z = 0$, and black (red) solid contours indicate $\delta z > 0$ ($\delta z < 0$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 5. (a) Topographies of the 4 craters selected from those shown in Fig. 4. The $y$-axis is in the strike direction (Fig. 1) and the origin $(x, y) = (0, 0)$ is defined at the point of largest negative displacement ($\delta z$) relative to the original surface. A black line indicates $y = 0$. (b) Crater profiles (red lines) at $y = 0$. Black broken lines indicate the origin surface. $\cdot$ indicate the largest negative $\delta z$ which defines the origin $(x = 0)$. (c) Displacement ($\delta z$) maps. Contours are drawn at 2 mm intervals. Black broken contours indicate $\delta z = 0$, and black (red) solid contours indicate $\delta z > 0$ ($\delta z < 0$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
slope steepens, the topographic minimum point within the crater floor shifts towards the downslope side, and the profile becomes asymmetric.

Here we classify the crater shape regimes I-III as a function of increasing \( \theta \). The craters shown in Fig. 5 are representative cases from each of these regimes. For the classification we use the following two criteria based on the quantitative measurement of topography: (i) using the crater profile (Fig. 5(b)), we determine whether a topographic minimum exists in the crater floor, and (ii) using the displacement map (Fig. 5(c)), we determine whether a crater rim, a region higher than the original surface, encircles the crater floor. For \( \theta = 0^\circ \) and \( 11^\circ \), the crater profile has a topographic minimum and a rim encircles the crater floor, and we define these as full-rim craters (regime I). For \( \theta = 22^\circ \), the crater profile has a topographic minimum but the rim disappears on its upslope side, and we define these as broken-rim craters (regime II). For \( \theta = 34^\circ \), the crater floor topography increases monotonically towards the upslope direction (i.e., there is no topographic minimum) and we define these as depressions (regime III).

We classify all the sand target experiments into the 3 regimes and summarize the results in the form of a regime diagram which we show in Fig. 6. Here the regimes are plotted in the parameter space of \( \theta \) and \( E \). This figure shows that regime I-II and II-III transitions occur at \( \theta \sim 20^\circ \) and \( \theta \sim 30^\circ \), respectively, and that within an order of magnitude variation of \( E \) in our experiments, the critical \( \theta \) for the regime transitions are approximately constant. For comparison we also indicate \( \theta = 16^\circ \) and \( \theta = 22^\circ \) for regime I and II, respectively. We find that the critical \( \theta \) for regime I-II and II-III transitions for the craters formed in sand and glass beads targets collapse to close values of \( \theta \sim 0.38 \) and 0.75, respectively, which are indicated by the broken vertical lines.

Using the profiles we measure the crater floor slope angles \( \psi_{us} \) and \( \psi_{ds} \), where the subscripts “us” and “ds” indicate upslope and downslope sides, respectively (see Fig. 9(a)). The details of the method used to measure \( \psi \) are as follows. First using the crater profile, we divide the crater floor into the upslope and downslope sides. When a topographic minimum exists within the crater floor, we define this point as the division point \( \psi \) in \( \theta = 0^\circ \) cases in Fig. 9(a)]. Otherwise we use the largest negative displacement \( \Delta z \) point as the division point (\( + \theta = 34^\circ \) case in Fig. 9(a)). We then calculate the slopes using the least squares method. For the fit we use more than 70% of the data between the crater edge and the division point, and choose the number of data so that the variance of error is minimized. Examples of the fits are shown in Fig. 9(a).
Fig. 8. Regime diagrams of sand (Fig. 6) and glass beads (Fig. 7(b)) targets plotted together as a function of normalized target slope $\hat{\theta}$ (Eq. (4)). Open and filled markers indicate the results for sand and glass beads targets respectively. Blue $\circ$, green $\triangle$, and red $\square$ indicate full-rim craters (I), broken-rim craters (II), and depressions (III), respectively. $\hat{\theta}_c = 1$ indicates slope at angle of repose. Magenta and cyan horizontal bars indicate slopes at collapse angles of sand ($\hat{\theta}_{c} = 0.544 - 0.627$) and glass beads ($\hat{\theta}_{c} = 0.489 - 0.595$), respectively, both decreasing with $E$. Broken vertical lines indicate the approximate regime I-II and II-III transitions at $\hat{\theta} = 0.38$ and 0.75, respectively. [For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.]

Fig. 9. (a) Profiles of craters formed in sand targets at slope angles of $\theta = 0^\circ$ and $34^\circ$ at an impact energy of $E = 0.63$ J. Green broken lines indicate the original surface. $\nabla$ indicate the topographic minimum and $\ast$ indicates largest negative displacement $\delta z$. Red (blue) broken lines indicate the linear fits of the crater floor slopes in the upslope (downslope) sides. Crater floor slope angles in the upslope (downslope) sides $\psi_{\text{fs}}$ ($\psi_{\text{ds}}$) are measured from the horizontal in the positive anticlockwise (clockwise) direction, as indicated by arrows. $\psi_{\text{fs}}$ is always positive ($\ast$), $\psi_{\text{ds}}$ is positive at $\theta = 0.16^\circ$ but becomes negative ($-$) at $\theta = 34^\circ$. (b) Normalized crater floor slope $\hat{\psi}$ (Eq. (9)) vs. normalized target slope $\hat{\theta}$ (Eq. (4)). Red (blue) markers indicate the crater floor slopes in the upslope (downslope) sides. Open (filled) markers indicate the craters formed in sand (glass beads) targets. Marker sizes increase with impact energy $E$. $\circ$, $\triangle$, $\square$ indicate regimes I, II and III, respectively. Vertical broken lines indicate the regime I-II and II-III transitions (see Fig. 8). $\hat{\theta}_c = 1$ indicates slope at angle of repose. Magenta (cyan) bar indicates slope at collapse angle ($\hat{\theta}_c$) of sand (glass beads). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
Here we normalize the crater floor slopes using the angle of repose \( \theta_r \) as

\[
\psi_{us,ds} = \left( \frac{\tan \psi_{us,ds}}{\tan \theta_r} \right),
\]

same as we did to normalize target slopes (Eq. (4)). Fig. 9(b) shows \( \psi_{us,ds} \) as a function of \( \hat{\theta} \) for both the sand (open markers) and glass beads (filled markers) targets. The figure shows that this normalization collapses the results for sand and glass beads targets fairly well. For full-rim craters (regime I), the crater floor slopes in the upslope and downslope sides are comparable (\( \psi_{us} \approx \psi_{ds} \)). However, for broken-rim craters (regime II), \( \psi_{us} \) increases, whereas \( \psi_{ds} \) decreases with \( \hat{\theta} \), which is evident from the diverge of red and blue markers in Fig. 9(b). For depressions (regime III), \( \psi_{us} \) asymptotically approach 1 whereas \( \psi_{ds} \) decreases and become negative (\( \psi_{ds} < 0 \)) with \( \hat{\theta} \), which corresponds to a monotonic increase of the crater floor topography towards the upslope direction (see \( \theta = 34^\circ \) case in Fig. 9(a)). We note that for depressions, the crater floor slopes are steeper than \( \theta_c \) for both in the upslope and downslope sides.

5.3. Crater formation process

Next we describe the results obtained from high-speed imaging. Fig. 10 and Movie 1 show the crater formation process in sloped sand targets. Images show that there are two important changes as the slope angle increases. First, the ejecta is dispersed farther towards the downslope side. Second, the avalanche towards the downslope side becomes significant such that the crater floor is buried. In the case of the steepest slope angle (\( \theta = 34^\circ \)), avalanche continues for a long time (\( \sim 0.3 \text{ s} \)) after the ejecta dispersal ends at \( \sim 130 \text{ ms} \), and buries most of the excavated crater floor at \( t = 470 \text{ ms} \). We also observe that multiple avalanches occur to form terraces.

Fig. 11 and Movie 2 show the crater formation process in sand viewed from a direction perpendicular to the target surface. A spherical impactor excavates the target at 5 ms, and is buried by the collapsing crater wall at 20 ms. The ejecta is dispersed farther towards the downslope direction (bottom in this figure) compared to the upslope direction, and as a result the ejecta curtain forms an elliptical ring.

Fig. 12(a) and Movie 3 show the images observed from the side for impact in sand target. These images show that as \( \theta \) increases,
the ejection becomes asymmetric such that the ejection angle is smaller and ejection velocity is larger towards the downslope direction compared to the upslope direction. Here we measure the time-averaged ejection angle $\phi$ and ejection velocity $v_e$ as follows. For each pair of images separated by a time interval of 1 ms, we subtract the former image from the latter image. We then binarize the images and track the edge of the ejecta curtain as a function of time. We use 5 data points (time interval 4 ms) immediately after the impact and obtain a linear fit using the least squares method. Examples of the fits superimposed on the last image used for the fits are shown in Fig. 12(a). From the fits we obtain $\phi$ measured from the horizontal and $v_e$, which are plotted as a function of $\theta$ in Fig. 12(b) and (c). Here we normalized $v_e$ using $v_{e0}$, which is the ejection velocity at $\theta = 0^\circ$. We could not accurately measure $\phi$ and $v_e$ towards the upslope direction for one sand target experiment at $\theta = 33.6^\circ$, and this data is not plotted. We conducted the same analyses for experiments in glass beads targets and the results are also plotted in Fig. 12(b), (c). These figures indicate that $\phi$ and $v_e$ become increasingly asymmetric as the target slope becomes steeper.

In Movie 4 we show the crater formation process in a glass beads target, indicating that the process is essentially the same as the sand target experiments. A difference is the ascent of a jet, which occurs simultaneously with the avalanche. As a result the jet is observed to translate downslope during jetting and finally stops at the lower end of the crater floor. This is the origin of the granular pile with a pair of dimples at its sides which can be seen in Fig. 7(a).

### 5.4. Definitions of 3 crater scales

Craters formed in sloped targets have asymmetric shapes and we use 3 length scales to characterize its shape. Fig. 13(a) shows a displacement ($\delta z$) map of a broken-rim crater formed in a sand target with a slope angle of $\theta = 16^\circ$ and Fig. 13(b) is its profile. The origin is at the location of largest negative $\delta z$ within the crater floor, and we define its vertical scale $|\delta z|$ as the crater depth $h_c$ (Fig. 13(b)). A broken line in Fig. 13(a) indicates a contour of $\delta z = 0$, which we define as the crater edge. A bounding box indicated in Fig. 13(a) is tangential to the crater edge and we define its two sides as the horizontal scales of the crater: the length $l$ and width $w$. Note that $l$ is the horizontal scale of the crater projected on the $x - y$ plane. Using $l$ and $w$ we define the crater aspect ratio $l/w$.

We note that when we define the 3 crater scales, we use the original surface as the reference which we did for the crater formed in a horizontal target (Fig. 3). We do not use the crater rim...
as the reference because part of the rim disappears for broken-rim craters and depressions. We also note that when we define the crater depth, we use the largest negative displacement relative to the original surface rather than the topographic minimum. Topographic minimum is not suited to define the depth scale of the depressions (regime III) because the topography increases monotonically towards the upslope direction.

5.5. Slope angle dependence of the crater scales and their ratios

Here we show how the 3 crater scales defined in the previous section, and their ratios, depend on the target slope angle \( \theta \), and derive fits which collapse the results for experiments using targets with different \( \mu_s \), \( \mu_d \) and different impact energies \( E \).

Fig. 14(a) shows the crater depth \( h_c \) as a function of \( \theta \) for both the sand and glass beads targets. The plot shows that \( h_c \) decreases with \( \theta \) as a consequence of the avalanche which buries the crater (Section 5.3). We also note that a deeper crater forms in a sand target which we described already for a horizontal target (Fig. 3). The increase of \( h_c \) with impact energy \( E \) is also evident. For comparison in this figure we also indicate the angle of repose \( \theta_r \) by crosses and the collapse angle \( \theta_c \) by horizontal bars, for both the sand and glass beads targets. The figure shows that \( h_c \) approaches zero as the slope angle increases towards \( \theta_c \). We also note that when \( \theta \) is larger than \( \theta_c \), \( h_c \) becomes less than a half of those of the craters formed in horizontal targets, indicating that the burial of the excavated transient crater becomes significant.

Here we normalize \( \theta \) and \( h_c \) so that the results for targets with different \( \mu_s \), \( \mu_d \) and impact energies \( E \) collapse to a single curve. To scale the effect of different \( \mu_s \) we use the normalized slope \( \hat{\theta} \) (Eq. (4)), same as we did in Fig. 9(b). To scale the effect of different \( \mu_d \) and \( E \), we normalize \( h \) using \( h_0 \) which is the depth of the crater formed in a horizontal target (\( \theta = 0^\circ \)) and at the same \( E \). The result of these normalizations is shown in Fig. 14(b) indicating a good data collapse which can be fit to a quadratic function (see figure caption for details).

Fig. 14(c) and (e) shows the crater length \( l \) and width \( w \) as a function of \( \theta \). These figures show that the crater becomes longer and wider as the slope steepens and as \( E \) increases. \( l \) and \( w \) are smaller for craters formed in sand because the crater wall collapse.
Fig. 15. (a) Crater depth $h_c$ / crater length $l$ vs. target slope angle $\theta$. Markers, crosses and horizontal bars are the same as those in Fig. 14. (b) Normalized version of (a). Vertical lines indicate the regime I-II and II-III transitions. A solid curve is a quadratic fit of $(h_c/l)/(h_c/l_0) = -0.358\theta^2 - 0.778\theta + 1.023$, where the subscript 0 indicates the ratio at $\theta = 0$. (c) Same as (a) but for crater length $l$ / crater width $w$ (aspect ratio). (d) Normalized version of (c) with a quadratic fit of $(l/w)/(l_0/w_0) = 0.180\theta^2 + 0.125\theta + 1.002$.

is limited. Normalized versions of these plots together with their quadratic fits are shown in Fig. 14(d) and (f). Comparing the $\theta$-dependence of $l$ and $w$, we find that $l$ depends more on $\theta$ than $w$, which is a consequence of $l$ being in the direction of the slope that is more affected by the avalanche. These figures indicate a good data collapse apart from a scatter for experiments in sand targets at steep slopes. At large $\theta$, craters formed in sand have rugged shapes and topographies caused by the avalanche (see $\theta = 34^\circ$ case in Fig. 10 and Movie 1) which continues after the crater excavation. As a result at large $\theta$, $l$ and $w$ of the craters formed in sand scale differently with $E$ compared to those at $\theta = 0^\circ$ (see Fig. 15(d)), which is a reason for a poorer data collapse.

We next proceed and calculate the two ratios between $h_c$, $l$ and $w$. First is the depth $h_c$ / length $l$ ratio which we plot as a function of $\theta$ in Fig. 15(a). The figure shows that $h_c/l$ decreases non-linearly with $\theta$ which is a combined result of crater becoming shallower (smaller $h_c$) and longer (larger $l$) as the slope steepens. $h_c/l$ is larger for craters formed in sand compared to craters formed in glass beads which corresponds to a steeper crater floor slope. This difference was already evident in craters formed in horizontal target (Fig. 3). We also note that at the same $\theta$, $h_c/l$ becomes smaller with $E$ indicating a smaller crater floor slope angle. In Fig. 15(b) we show a normalized version of Fig. 15(a) where we similarly normalize $h_c/l$ by $(h_c/l)_0$, which is the ratio at $\theta = 0^\circ$ for respective $E$. We fit these data to a quadratic function

$$\left(\frac{h_c/l}{(h_c/l)_0}\right) = a\left(\frac{\tan \theta}{\tan \theta_0}\right)^2 + b\left(\frac{\tan \theta}{\tan \theta_0}\right) + c,$$

and the prefactors $a$, $b$, $c$ are given in the figure caption. Second is the length $l$ / width $w$ ratio or the aspect ratio, which characterizes the asymmetry of the shape when viewed from above. Fig. 15(c) shows $l/w$ as a function of $\theta$, and Fig. 15(d) shows its normalized version. This figure shows that $l/w$ increases with $\theta$, indicating that the crater becomes increasingly elongated in the direction of the slope. We also note a large scatter of $l/w$ for experiments in sand target, which is a result of rugged crater shape at steep slopes.

5.6. Slope angle dependence of the energy scaling

Fig. 16(a)–(c) shows the impact energy ($E$) dependence of the crater depth $h_c$, length $l$, and width $w$. Here the measurements for different slope angles $\theta$ are indicated by the different colors. The range of $E$ in our experiments is limited to 1 order of magnitude but we tentatively fit these data to a power law function ($\propto E^n$) and obtain the power law exponents $n$ (error $\delta n$) for each $\theta$. Fig. 16(d) shows the obtained $n \pm \delta n$ plotted as a function of normalized slope $\tilde{\theta}$ (Eq. (4)). For reference we indicate $n = 0.25$, which is the exponent for gravity regime scaling (Holsapple, 1993). In Fig. 16(d), $\delta n$ are indicated by error bars, and the relative errors are $\delta n/n = 0.05 - 0.12$ for sand and $\delta n/n = 0.02 - 0.08$ for glass beads. These values are quite small indicating a good fit to a power law function despite our limited $E$ coverage.

First we describe the results of $n$ for a horizontal target ($\tilde{\theta} = 0$). Fig. 16(d) shows that $n$ of the crater length $(\triangleright)$ and width $(\equiv)$ for both the sand (red markers) and glass beads (blue markers) targets are in the range of $n \simeq 0.23 - 0.25$ and are close to $n = 0.25$. On the other hand $n$ of the crater depth $(\triangleright)$ are in the range of $n \simeq 0.13 - 0.18$ and are $< 0.25$. This implies that the crater depth / length ratio decreases with $E$, which we showed in Fig. 15(a). Next we describe the results for sloped targets ($\tilde{\theta} > 0$). Fig. 16(d) shows that there are no systematic dependences of $n$ on $\tilde{\theta}$. However we find that for $\tilde{\theta} \geq 0.61$, $n$ of sand targets deviate from those
at smaller $\theta$, and have larger errors. This is a consequence of a rugged crater shape caused by the avalanche. We also find that similar to the results at $\theta = 0$, $n$ of the crater length and width are larger than $n$ of the crater depth, for all but $\theta \approx 0.61$ cases for the sand target. To summarize, the power law exponents $n$ of the crater scales - impact energy relations are little affected by the target slope, apart from those at steep sand slopes.

We note that we may alternatively plot the results shown in Fig. 16(a)–(c) in a dimensionless form (e.g., crater scales / $D$ vs. $E/\phi$). For a gravity regime scaling, either versions of the plots yield $n = 0.25$. Previous experiments have shown that crater scales vs. $E$ plot collapses the results for different $D$ (Amato and Williams, 1998; Uehara et al., 2003; Takita and Sumita, 2013). We choose to plot in a dimensional form following these works, and also because the error of the fits ($\delta n/n$) are smaller compared to the dimensionless version.

6. Discussions

6.1. Comparison with previous experiments

First we compare with previous low-velocity impact cratering experiments conducted in horizontal granular targets. It is well established that the power law exponent $n$ of the crater diameter - energy scaling relation is $n \approx 0.25$ (Amato and Williams, 1998; Uehara et al., 2003; Walsh et al., 2003; Takita and Sumita, 2013). It is also known that crater depth / diameter ratio decreases with impact energy (Walsh et al., 2003; de Vet and de Bruyn, 2007; Takita and Sumita, 2013). Our results at $\theta = 0^\circ$ are consistent with these works. Here we compare with the results of Takita and Sumita (2013) in further detail. They conducted experiments using the same sand but loosely packed ($\phi = 0.53$), at an impact energy of $E \approx 0.01 - 0.73$ J which covers a wider $E$ range compared to our experiments ($E = 0.055 - 0.63$ J). When we compare the power law exponents $n$ we obtained for the crater length, width - $E$ relation (red $\circ$ and $\square$ at $\theta = 0$ in Fig. 16(d)), with $n$ they obtained for the crater diameter - $E$ relation (black $\bigcirc$), we find that the $n$ are well reproduced despite a narrower $E$-range. Similarly comparing $n$ we obtained for the crater depth - $E$ relation (red $\triangledown$ at $\theta = 0$) with $n$ they obtained (black $\triangledown$), we find that it is also well reproduced. However at the same $E$, craters formed in our experiments are narrower and deeper (compare black $\bigcirc$ with gray broken lines in Fig. 16(a)–(c)), and therefore have a steeper crater floor slope. In addition we did not observe jet and central peak formation, which occurred at $E \geq 0.2$ J in the experiments of Takita and Sumita (2013). It seems that these differences are results of our using a more closely packed ($\phi = 0.57$) sand, which suppresses the crater wall collapse and jet formation.

Next we compare our ejection angle $\phi$ and velocity $v_e$ measurements with those of Deboeuf et al. (2009) who conducted experiments at comparable parameters ($v_i = 1 - 4$ m/s, $E = 0.003 - 0.2$ J), and studied how the shape of a granular corona, a largest visually connected grains when viewed from the side, evolves with time. They showed that the height-time data of the corona fit well to a parabolic flight path, and from the fit they estimated $\phi$ and $v_e$. This method differs with ours since we tracked the edge of the ejecta curtain regardless of whether they form largest connected grains. They obtained $\phi \approx 53^\circ$ which is larger than $\phi \approx 40 \pm 7^\circ$ which we obtained in our experiments at $\theta = 0^\circ$. Their obtained $v_e$ normalized by the impact velocity $v_i$ was $v_e/v_i \sim 0.3$, which
is smaller than our result of \( v_e / v_f \approx 0.73 \pm 0.25 \) at \( \theta = 0^\circ \). We consider that their measurements give a smaller \( v_e \) because they tracked the largest connected grains.

Third we compare with oblique impact cratering experiments which also form asymmetric craters (e.g., Gault and Greely, 1978). Wang and Zheng (2013) conducted low-velocity oblique impact cratering experiments and showed that as the impact angle from the horizontal becomes smaller, the crater becomes shallower and elongated in the direction of impact. These crater shapes are qualitatively similar to those formed in slopes. However the mechanism for the formation of asymmetric craters is different. For oblique impacts, asymmetric craters form because horizontal impact momentum becomes dominant. In contrast for impact in slopes, asymmetry is caused by the avalanche towards the downslope direction. They also showed that when the impact angle from the horizontal becomes \( < 30^\circ \), the crater length / width ratio increases to \( \sim 2 \), and the power-law exponents \( n \) of the crater length - impact energy relation increases to \( n \sim 0.45 \), which deviates from \( n = 0.25 \) for the gravity regime scaling. Such a significant elongation and deviation of \( n \) were not observed in our experiments (Figs. 15(c), 16(d)).

Finally we compare with previous experiments on sloped granular targets. Furbish et al. (2007) conducted rain splash experiments and studied the statistics of grains ejected in different directions and distances from the impact point and showed that more grains are ejected farther downslope. Aschauer et al. (2016) measured the aspect ratio of impact craters, same as our length / width, and showed that it increases with the slope angle. They also showed that when viewed from the side, ejection is asymmetric. Our results are consistent with their findings.

6.2. Modelling ejection asymmetry

Maxwell’s Z-model (Maxwell, 1977) is a simple kinematic model of crater excavation flow. The ejection angle and velocity predicted by this model is known to agree reasonably well with those measured for impact and explosive cratering on a horizontal target (Melosh, 2011). The plausibility of Z-model has also been confirmed from numerical simulation of granular impact cratering (Wada et al., 2004). By combining the Z-model with a ballistic model after ejection, it is possible to calculate the whole trajectory during excavation and ejection. Here we apply these models to cratering on slopes.

The flow in the Z-model is conventionally expressed in polar coordinates \((R, \theta)\), where \( R \) is the radial distance and \( \theta \) is the polar angle measured from the vertical downward direction. The flow source is at the origin \((R = 0)\) and the radial velocity component is assumed to decrease with \( R \) as

\[
  u_R = \frac{\alpha}{R^2}.
\]

(11)

Here \( \alpha \) and \( Z \) parameterize the flow strength and pattern, respectively. From continuity the angular (positive upward) velocity component is expressed as

\[
  u_\theta = (Z - 2) \tan \left( \frac{\theta}{2} \right) \cdot u_R.
\]

(12)

For \( Z > 2 \) the flow is deflected upwards towards the surface. Streamlines are expressed as

\[
  R = R_0 (1 - \cos \theta)^{1/(Z-2)},
\]

(13)

where \( R_0 \) is a constant which is different for each streamline.

Here we apply the Z-model to ejection at slopes to account for the difference of the elevation of the flow source and the ejection point. Similar modification has been made by Croft (1980). For a slope angle \( \theta_i \), \( \theta_f \) at the ejection point is

\[
  \theta = 90^\circ \pm \theta_i.
\]

(14)

where + and - indicate upslope and downslope sides, respectively. From Eqs. (11),(12), the ejection angle \( \phi \) measured from the horizontal is expressed as

\[
  \phi_e = \arctan \left( \frac{Z - 2}{(1 - \cos \theta_i)^{1/(Z-2)}} \right) \pm \theta_i.
\]

(15)

For a horizontal target \((\theta_i = 0^\circ)\), \( Z = 3 \) results in an ejection angle of \( \phi = 45^\circ \). Using Eq. (15), we calculate \( \phi_e \) as a function of \( \theta_i \) using \( Z = 2.5 \) and \( Z = 3.3 \), which we show in Fig. 12(b). Comparing with the measurements we find that although the model explains the general trend, it predicts a larger \( \theta \)-dependence of \( \phi_e \).

Ejection velocity \( v_e \) can be calculated from Eqs. (11),(12) as

\[
  v_e = \sqrt{u_R^2 + u_\theta^2}.
\]

(16)

where the velocity components are calculated at the ejection point. In Fig. 12(c) we show the normalized ejection velocity \( v_e / v_{e0} \), where \( v_{e0} \) is \( v_e \) at \( \theta = 0^\circ \). We find that \( v_e / v_{e0} \) calculated from the model agree reasonably well with the measurements in the upslope side, but overestimate in the downslope side. We note that the Z-model assumes a point source and an incompressible steady flow, and we assumed \( \alpha \) and \( Z \) as constants. Our image analyses were conducted during the initial stage of ejection, and therefore the effect of finite impactor size and time-dependence may have caused these discrepancies.

Fig. 17 shows the trajectories calculated by combining the Z-model in the target (solid line) and a ballistic model after ejection (broken line). Here we assumed \( Z = 2.9 \) which explains the \( \theta \)-dependence of the ejection angle (Fig. 12(b)) reasonably well, and chose \( \alpha \) so that \( u_e = 1.01 \) (m/s) at \( R = R_0 = 0.01 \) (m) (Eq. (11)) to model the experiments. The figure shows that for a sloped target, ejecta is dispersed farther towards the downslope side due to a larger horizontal component of ejection velocity and a longer time until sedimentation. Together with the overall avalanche which occurs after the excavation and erodes the upslope side, this provides an explanation for the formation of a thick rim in the downslope side.

6.3. Comparison with the craters on the slopes of Vesta

First we estimate the dimensionless numbers \( g' \) (Eq. (5)) and \( i' \) (Eq. (6)) for impact cratering on Vesta. Yield stress (strength)
Fig. 18. (a) Crater profiles on Vesta (black line) superimposed on a crater formed in our experiments (red line) with a scale factor of 10⁷. A red broken line indicates the original surface of a laboratory crater. Vesta crater is from Fig. 5a (Oppia) in Krohn et al. (2014a). Laboratory crater formed in a sloped sand target (θ = 11°) at an impact energy of E = 0.58 J. The origins of the horizontal and vertical coordinates are chosen at the deepest point in the crater floor. Vertical and horizontal scales for the Vesta (laboratory) crater profiles are indicated on the left (right) and bottom (top). The horizontal range is × 5.6 that of the vertical range for both profiles. (b) Same as (a) but for Fig. 3b in Krohn et al. (2014a) and a laboratory crater formed in a sloped (θ = 26°) sand target at E = 0.095 J. Scale factor is × 6.7 × 10⁵. The horizontal range is × 2.5 that of the vertical range. (c) Crater depth hₑ / length l vs slope angle θ of 9 Vesta crater profiles from Krohn et al. (2014a) and those of our experiments (same as Fig. 15(a)). Vesta craters are plotted in * and their sizes correspond to hₑ (hₑ = 0.96 km − 9.08 km, see text for the measurement method). Curves are calculated from Eq. (19) with two frictional parameters μₛ and μ₅ (see legend). × indicate the corresponding angles of repose θₛ. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

σᵧ of the regoliths or rocks consisting the surface of Vesta is unknown. Compilation of σᵧ for geological materials show that σᵧ can vary by 2 orders of magnitude, ranging from σᵧ ∼ 0.18 MPa for dry soil, σᵧ ∼ 8 MPa for soft rocks to σᵧ ∼ 18 MPa for hard rocks (Holsapple, 1993). Here we assume σᵧ = 8 MPa, an intermediate value. Depth hₑ of the craters on Vesta vary in the range of hₑ = 0.1 − 10 km (Vincent et al., 2014). We hence assume a relatively deep crater with a depth of hₑ = 3.3 km. This is an average depth of the 9 craters (hₑ = 0.96 − 9.08 km) whose profiles are reported in Krohn et al. (2014a) and which we compare in detail. Substituting these values we obtain

\[
g′ \sim 0.3 \left( \frac{8 \text{ MPa}}{\sigmaᵧ} \right) \left( \frac{hₑ}{3.3 \text{ km}} \right). \tag{17}\]

and

\[
l′ \sim 45 \left( \frac{8 \text{ MPa}}{\sigmaᵧ} \right). \tag{18}\]

Here we used ρₛ = ρₑ = 2800 kg/m³ for the density of the target and impactor, a surface gravity of g = 0.25 m/s², and an escape velocity of vₑ = 360 m/s (Russel et al., 2012). Above estimate indicates that g′ of deep (hₑ > 1 km) craters on Vesta may be comparable to g′ = 0(0.1 − 1) (Section 4) of our experiments, indicating that it is in a gravity regime or in the transition of gravity and strength regimes.

Next we compare with the 9 crater profiles in Krohn et al. (2014a), which appear to correspond to the full-rim or broken-rim craters in our experiments. In Fig. 18 (a), (b), we superimposed scaled crater profiles formed in our experiments with those on Vesta. The profiles nearly overlap although their scales differ by 5 - 6 orders of magnitude, suggesting a common formation process and a small degradation. We measured the slope angle, depth, and length of 9 craters on Vesta and calculated the depth / length ratios. Here we estimated the original surface on Vesta by connecting the end points of the crater profiles, and measured the depth and length of the craters using the same method which we used to analyze the experiments. For some craters the edge of the crater in the upslope or the downslope sides could not be defined because the crater edge were below the assumed original surface. However for such craters, we could define the point at which the depth was at a local minimum, and we defined the crater edge accordingly. Fig. 18(c) shows the results, which are compared with the experiments.

In Fig. 18(c) we also indicate the fit (Eq. (10)) between the depth hₑ / length l ratio and the target slope angle θ. Using the prefactors obtained from the fit, and the definitions Eq. (1),
\[ \frac{h_t}{l} = \frac{\mu_d}{2} \left( -0.358 \left( \frac{\tan \theta}{\mu_s} \right)^2 - 0.778 \left( \frac{\tan \theta}{\mu_s} \right) + 1.023 \right). \]  

The curves for the experiments in sand and glass beads targets are drawn using their respective values of \( \mu_s \) and \( \mu_d \). A black broken curve is calculated using \( \mu_s = 0.680 \) and \( \mu_d = 0.340 \), which are values between those of sand and glass beads. This value of \( \mu_d \) corresponds to \( (h_t/l)_0 = 0.170 \) at \( \theta = 0^\circ \). Analyses of Vestan craters have shown that the distribution of depth / diameter ratio peaks at 0.168 ± 0.01 (Vincent et al., 2014), which is close to the above \((h_t/l)_0\) value. From this figure we find that the depth / length ratio of 7 out of 9 Vestan craters are plotted between the curves of sand and glass beads targets. This suggests that the crater formation in these Vestan craters can be approximated by a granular flow with an effective friction coefficient between those of sand and glass beads. However for this limited number of data, we do not find a decreasing trend of \( h_t/l \) with \( \theta \). This may be partly due to the error of the estimated original surface and also to the variation of \( \mu_s \) and \( \mu_d \) depending on the geology of the impact site.

We remark that there are features of Vestan craters which are not reproduced in our experiments, such as the presence of a separation line at the border of the upslope and downslope sides of the crater floor, and crater elongated in the strike direction rather than in the slope direction (Jaumann et al., 2012; Krohn et al., 2014a). These features may have resulted from the conditions and processes which were not modelled in our experiments. The separation line may have formed from mass wasting within the crater (Krohn et al., 2014b) triggered by other impacts. Crater elongation in the strike direction may have been caused by oblique impacts or from inhomogeneous target strength. Other crater degradation effects such as ejecta sedimentation from other impacts and the effects of the non planar target surface may also result in craters whose shapes differ from those formed in our experiments.

7. Conclusions and final remarks

Impact cratering experiments in granular slopes show that diverse crater shapes form as the slope angle \( \theta \) steepens. These crater shapes result from asymmetric ejecta dispersal and overall avalanche towards the downslope direction. Using a normalized slope \( \theta_1 \), we obtained fits for the \( \theta_1 \)-dependence of crater depth, length, width and their ratios. Since these fits scale the effects of \( \mu_s \) and \( \mu_d \), it can be applied to targets with arbitrary effective friction coefficients. When we compare the craters we formed in our experiments with the deep craters on Vesta, we find that some have remarkably similar profiles and many have depth / length ratios which are close. This suggests that these Vestan craters also formed in the gravity regime and that the cratering process can be approximated by a granular flow with similar effective friction coefficients.

Finally we note that cratering on slopes is not restricted to asteroids or small satellites, but may also occur in larger planets. One situation is when an impact occurs in a preexisting larger crater. In such situation, such as the crater floor slope angle is close to the collapse angle \( \sim \theta_c \approx 0.5 \text{ to } 0.6 \), our regime diagram (Fig. 8) suggests that broken-rim crater (II) forms having a depth / length ratio about a half of those formed at \( \theta = 0 \) (Fig. 15(b)). Another situation is when an erosion occurs on the slope of a volcano. One example is a crater formed in the south east slope \( \approx 17^\circ \) of Fuji volcano (Miyaji et al., 2011). This crater has an asymmetric shape, elongated in the direction of the slope, similar to the impact craters formed in our experiments.

Acknowledgments

We thank K. Krohn for kindly providing us with the crater profile data of Vesta, and two anonymous reviewers for the comments which helped to improve the manuscript. One reviewer suggested an accurate method to calculate \( \mu \) from rheometry. This work was supported by JSPS, KAKENHI Grant Numbers 24244073, 24510246 and the T. Ishihara Research Scholarship from Association for Disaster Prevention Research, Kyoto, Japan.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.icarus.2017.03.027

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