A laboratory model for melting erosion of a magma chamber roof and the generation of a rhythmic layering
Yasuko Shibano,1 Ikuro Sumita,1 and Atsuko Namiki2
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[1] A hot magma chamber can ascend by melting the roof rock, a process which in turn affects the magma composition. The disaggregated mineral particles which consisted the roof rock will descend in the magma chamber to form a sedimentary cumulate. However, the fluid dynamics leading to the formation of the sediment, and how we can decipher them is unknown. Here we conducted a series of experiments modeling melting erosion of the roof with particle size consisting the roof rock as the parameter. We find that there is a critical particle size below which the melting erosion occurs rhythmically. Melting erosion stops because the disaggregated particles are suspended in the magma chamber and suppress the vertical heat transfer. The suspension then separates into an upper clear layer and a lower suspension layer. Eventually, the heated stratified layers become unstable. An overturn occurs, and melting erosion resumes. When the particles consist of two sizes such that at least one of them is smaller than the critical size, a rhythmic erosion occurs. Particles are sorted during each erosion period, and a size-graded rhythmic layering is spontaneously generated. We estimate that rhythmic layering can be generated from melting erosion in a basaltic magma chamber when the grain size of the roof rock is ≤ 0.6 mm, assuming a vertical temperature difference of 10°C. We suggest that rhythmic roof melting coupled with particle settling is one possible mechanism for generating the rhythmic layering which is commonly observed in solidified magma chambers.


1. Introduction

[2] A hot magma which intrudes into the host rock can melt its roof and ascend upward (Figure 1a). This mechanism of magma ascent is likely to become important at high temperatures and pressures where dykes do not form [e.g., Harris, 1957; Kushiro, 1968; Philpotts and Ague, 2009]. In a vigorously convecting magma chamber, the vertical temperature profile is close to an adiabat, which has a smaller temperature gradient compared to the solidus or the liquidus gradients of the magma and the host rock, an example of which is shown in Figure 1b (see section 4 for details). As a result, melting of the host rock occurs at the roof and solidification of the magma occurs at the bottom [Harris, 1957; Kushiro, 1968; Ahern et al., 1981; DePaolo, 1981; Oxburgh and McRae, 1984; Philpotts and Ague, 2009]. The latent heat released from solidification at the bottom of the magma chamber provides an additional heat source to melt the roof rock. This mechanism can assimilate the molten roof rock and concentrate incompatible elements into the magma [e.g., DePaolo, 1981], to generate magmas with a wide range of composition, which cannot be explained by fractional crystallization alone. In addition, for a typical magma chamber size ~ < 10 km, ascent by roof melting can become faster than ascent by deforming the host rock (i.e., by diapirs) (see section 8.3, Figure 11).

[3] If the roof rock becomes only partially molten (Figure 1a), as the melt fraction increases, the mineral particles will disaggregate [e.g., King et al., 2011] and settle downward [e.g., Kushiro, 1968], a process analogous to stopping [e.g., Daly, 1903; Marsh, 1982; Furlong and Myers, 1985; Marsh, 1989]. The settled particles will form a sedimentary cumulate at the bottom of the magma chamber, which remain solid because the melting temperature is higher than that at the roof [e.g., Philpotts and Ague, 2009]. Indeed, field observations show that fragments of roof rock are occasionally present in the layered intrusions [e.g., Irvine, 1965, 1974; Irvine et al., 1999, 2005]. Kushiro [1968] called magma ascent by partially melting the roof rock, partial zone melting, and argued that this mechanism can explain the compositional change of magma as it ascends. This concept has been applied to
interpret geological observations [Arai et al., 1988]. However, the consequences of the particle settling from the magma chamber roof on the subsequent dynamics within the magma chamber are hardly understood.

[4] When a basaltic magma intrudes into a host rock, a partially molten roof can form under various geological settings. One situation is intrusion into a more mafic host rock. Here the liquidus temperature of the roof rock is higher than that of the basalt, as shown in Figure 1b. Such situation should be realized in Earth and may occur within the mantle [e.g., Kushiro, 1968] or at the crust-mantle transition zone [e.g., Kelemen et al., 1997]. For the case in which the more mafic host rock is the mantle rock, the liquidus temperature is higher than that of the basalt by as much as \( \sim 500^\circ C \) (see Figure 1b and captions). Accordingly, even when a superheated (i.e., having a temperature in excess of the liquidus) basalt intrudes, the roof rock becomes only partially molten and the particles consisting the roof rock will settle downward. As the magma cools, crystals will also form and settle in the magma chamber. Consequently, the mineral particles consisting the cumulate will have two origins; those from the roof (restites), and those from the crystallization of the basaltic magma [e.g., Huppert and Sparks, 1988a].

[5] Partially molten roof can even form when the basaltic magma underplates or intrudes into a crust having a similar composition, for example, at the roof of the oceanic ridge magma chamber [e.g., Gillis and Coogan, 2002; Coogan et al., 2003], or a more silicic composition, for example, at the continental crust [e.g., Huppert and Sparks, 1988a; Bergantz, 1989]. Here the solidus and liquidus temperatures of the roof rock are comparable to or lower than those of the basaltic magma. Therefore, when the temperature of the magma is higher than the roof rock liquidus, the roof rock in contact with the magma will completely melt. However, with cooling, the temperature of the magma becomes lower than the roof rock liquidus, and the roof rock becomes partially molten. Under such temperature condition, the refractory minerals consisting the roof rock will settle downward [Huppert and Sparks, 1988a].

[6] There are many dynamical processes which accompany roof melting (Figure 1a). Descending particles can form plumes which drive flow [Simakin et al., 1997; Michioka and Sumita, 2005; Shibano et al., 2012] be suspended within the magma [Marsh and Maxey, 1985; Weinstein et al., 1988; Martin and Nokes, 1988, 1989; Lavorel and Le Bars, 2009], advect heat [Blanchette et al., 2010], suppress thermal convection [Koyaguchi et al., 1990, 1993], and be reentrained from the sediment [Solomatov et al., 1993]. The coupled result of these effects is poorly known.

[7] Several laboratory experiments and their theoretical analyses have been conducted to understand the fluid mechanics associated with roof melting [e.g., Campbell and Turner, 1987; Huppert and Sparks, 1988a, 1988b; Kerr, 1994; Kaneko and Koyaguchi, 2000; Leitch, 2004]. Magma ascent by roof melting and the simultaneous cooling of the magma have been numerically simulated, which showed that a 5 km-sized basaltic magma chamber can ascend \( \sim 2km \) [Ahern et al., 1981]. Recently, the mechanism of the reaction of a silicic crystalline mush by a magma intruding from the bottom has been examined in detail [Huber et al.,

Figure 1. (a) Melting erosion of a roof rock in a thermally convecting magma chamber and the accompanying processes. (b) A schematic diagram of a vertical temperature profile in a basaltic magma chamber and the heated host rock (thick red line), and its relation to the solidus and liquidus temperatures, and the geotherm (host rock temperature before magma intrusion). \( \Delta T \) is the superheating (temperature difference between the adiabat and the basalt liquidus) at the top of the magma chamber, \( H \) is the superliquidus height range, \( \delta T \) is the temperature rise needed for the host rock to reach basalt liquidus. This figure corresponds to the case in which the magma intrudes into a host rock having a higher liquidus temperature, similar to the experiments. Other situations are also possible (see text). Basaltic magma and a more mafic roof rock (e.g., peridotite) have similar solidus temperatures of \( T_{solid} \approx 1100^\circ C \) at \( \sim 10^5 \) Pa (1 bar) [Hess, 1989; Herzberg, 1995]. However, liquidus temperature of a basalt is \( T_{liquid} \approx 1200^\circ C \) [Hess, 1989] whereas that of the roof rock can be as high as \( \sim 1700^\circ C \) [Herzberg, 1995]. (c) Experimental set up. \( \phi_0 \) and \( \phi(t) \) are the particle volumetric fraction in the particles + solid wax layer and in the molten wax layer, respectively. \( V_m \), \( V_e \), \( V_i \), and \( V_c \) are melting, erosion, Stokes settling, and convection velocities, respectively.
2010, 2011; Burgisser and Bergantz, 2011]. Flow and mixing resulting from intrusion have also been simulated [Gerya and Burg, 2007; Simakin and Bindeman, 2012]. Although it has been recognized that particle settling can occur as a result of partial melting of the roof rock [e.g., Kushiro, 1968; Ahern et al., 1981; Huppert and Sparks, 1988a], there is only a limited experiment [Campbell and Turner, 1987] which modeled this situation. This lack of study requires a more detailed investigation. In this paper, we report the results of a series of experiments which focus on understanding the basic physics of the coupled result of roof melting and particle settling in a thermally convecting magma chamber. The experimental results are then scaled to the case in which a basaltic magma intrudes into a more mafic host rock.

2. Experimental Methods

[8] Experiments are conducted in a thin square cell (Figure 1c). We use a wax (PEG 1000) which melts at 37°C and glass beads (hereafter particles), to model the intruding magma (e.g., basalt) and the refractory mineral particles of the roof rock (e.g., olivine), respectively. Physical properties of wax and glass beads are summarized in Tables S1–S3. A rectangular silicone rubber heater (130×25 mm : Samicon Super 340, Sakaguchi) was attached to the bottom of the copper base plate and a constant temperature was maintained by a thermostat (Omron). Temperatures in the experimental cell were measured by K-type thermocouples (diameter 0.65 mm) with welded ends, and time-series data were recorded every 3 s. The total number of thermocouples inserted into the cell is either 0, 13, 14, or 20 and is positioned at vertical intervals of ≥ 3 mm. Temperatures of the bottom and top copper plates and the room temperature were also measured during the experiments. We used two types of lightings. When the temperature field is visualized by thermotropic liquid crystals (Japan Capsular), we illuminate the cell from the side using a halogen light sheet (Moritex). Otherwise, we illuminate the cell from the front using a fluorescent lamp.

[9] The experimental procedure is as follows. The sample was prepared by thoroughly mixing the glass beads and the molten PEG wax, to which a small amount of surfactant was added to eliminate the trapped air bubbles. The mixture was allowed to settle and compact within the molten wax, before allowing the wax to solidify completely. We then invert the cell and obtain the initial condition with an upper layer (thickness : h = 32.3 ± 2.0 mm) of a mixture of particles and solid wax (particle volumetric fraction φ0 = 0.49 ± 0.08), overlying a lower layer (thickness : H = 47.7 ± 2.0 mm) of solid wax (Table 2).

[10] The cell was heated from below at a fixed temperature of 69.9°C (standard deviation 0.02°C), and the wax melts. The top and side walls are at room temperature (25 ± 1°C). The molten wax layer thickens with time (Figure 2b). When the wax within the particles-wax layer starts melting, particles disaggregate and settle. We continue to heat the cell from below, which models the latent heat released by solidification at the bottom of the magma chamber (Figure 1a).

3. Dimensionless Numbers

[11] The dimensionless numbers relevant to our experiments and in the magma chamber are summarized in Table 1. Thermal convection is characterized by a Rayleigh number

$$Ra = \frac{\alpha g \Delta T h^3}{\nu \kappa},$$  (1)

where α is the thermal expansivity, g is the gravitational acceleration, ΔT is the vertical temperature difference across the melt layer, κ is the thermal diffusivity, ν is the kinematic viscosity (see Table S1 for physical properties of molten wax and basaltic melt), and a Prandtl number

$$Pr = \frac{\nu}{\kappa}.$$  (2)

[12] The high Ra (≫ Ra*): critical Rayleigh number ≃ 1708, Chandrasekhar [1961]) and Pr (≫ 1) numbers of our experiments are appropriate for modeling the convection in the magma chamber. Here we note that the Ra and Pr defined above are for the case in which the melt does not contain suspension. Stefan number is defined by

$$St = \frac{L}{C_0 \delta T},$$  (3)

where L is the latent heat, C is the specific heat, and δT is the temperature rise needed for melting and is the ratio of the latent to sensible heats. These two heats originate from the convected heat and correspond to those which are used to melt and warm the roof rock at the top of the magma chamber. They also correspond to the two terms which affect the melting velocity of the roof (equation (7)). Reynolds number is defined by

$$Re = \frac{V H}{\nu},$$  (4)
within 2 orders of magnitude (Table 1). and the magma chamber are of the order of 0.1–10, and agree the magma chamber becomes superheated. We use the super-
vecting part of the magma chamber, the temperature profile can be considered to be adiabatic (Figure 1b). Because the temperature difference which drives the thermal convection is defined by the melting temperature of wax \( T_{\text{liq}} \) which is the temperature close to the basalt liquidus at the top of the magma chamber. How-
thermal boundary layer, which we evaluated from the thickness. According to the estimate of equation (5), \( \Delta T \) scales in proportion to the magma chamber thickness. For a magma chamber thickness of \( H = 4 \) km, which is close to the largest magma chamber thickness considered by Ahern et al. [1981], \( \Delta T \) becomes \( \Delta T \approx 10^6 \) C. We note that our estimate for \( \Delta T \) is simplified because we consider the superheated part of the magma chamber only. In detail, the simplifications are as follows. First, the lowermost part of the vigorously convecting part of the magma cham-
becomes subliquidus having a two-phase adiabat (see Solomatov [2007] for a review), but this is not included in \( H \). Second, we considered \( \Delta T \) as the temperature drop across the top thermal boundary layer, which we evaluated from the temperature difference between the convecting magma and the basalt liquidus at the top of the magma chamber. How-
value can be modified by the disaggregation of the particles consisting the roof rock.
where \( V_c \) is the convection velocity, and compares the inert-
both \( Sr \) and \( Re \) of the experiments and the magma chamber are of the order of 0.1–10, and agree with 2 orders of magnitude (Table 1).

### 4. Characteristic Temperature Difference Scales

[13] There are two temperature difference scales relevant to our experiments. First is the vertical temperature difference \( \Delta T = 33^\circ \text{C} \) between the bottom plate \( 70^\circ \text{C} \) and the temperature of the solid wax-liquid wax interface which is defined by the melting temperature of wax \( 37^\circ \text{C} \). This is the temperature difference which drives the thermal convection in the molten wax, when there is no sediment. Second is the temperature difference \( \delta T = 12^\circ \text{C} \) which is the temperature rise needed for the solid wax at room temperature \( 25^\circ \text{C} \) to start melting at \( 37^\circ \text{C} \).

[14] We similarly estimate the corresponding temperature difference scales in the magma chamber. As one example, we consider a situation in which a basaltic magma intrudes into a more mafic host rock. Intrusion occurring in other geological settings is also possible (see section 1) for which case the estimated values will differ accordingly. First, we consider \( \Delta T \) in the magma chamber. In a vigorously convecting part of the magma chamber, the temperature profile can be considered to be adiabatic (Figure 1b). Because the liquidus temperature gradient is steeper than the adiabat, the magma chamber becomes superheated. We use the superheating at the top of the magma chamber for \( \Delta T \), which can be estimated from

\[
\Delta T = H \frac{dT_{\text{liq}}}{dz} = 2.6(\text{C}) \left( \frac{H}{1 \text{ (km)}} \right),
\]

where \( H \) is the height range in which the magma chamber is superheated, \( dT_{\text{liq}}/dz \) is the basalt liquidus temperature gradient, and \( dT_{\text{ad}}/dz \) is the adiabatic temperature gradient. Here we used \( dT_{\text{liq}}/dz \approx 3 \text{K km}^{-1} \) [Hess, 1989] and \( dT_{\text{ad}}/dz = a g T/C \approx 0.4 \text{ K km}^{-1} \). A similar method for estimating \( \Delta T \) has been made by Ahern et al. [1981]. We hereafter use \( H \) to approximate the magma chamber thickness. According to the estimate of equation (5), \( \Delta T \) scales in proportion to the magma chamber thickness. For a magma chamber thickness of \( H = 4 \) km, which is close to the largest magma chamber thickness considered by Ahern et al. [1981], \( \Delta T \) becomes \( \Delta T \approx 10^6 \) C. We note that our estimate for \( \Delta T \) is simplified because we consider the superheated part of the magma chamber only. In detail, the simplifications are as follows. First, the lowermost part of the vigorously convecting part of the magma cham-
becomes subliquidus having a two-phase adiabat (see Solomatov [2007] for a review), but this is not included in \( H \). Second, we considered \( \Delta T \) as the temperature drop across the top thermal boundary layer, which we evaluated from the temperature difference between the convecting magma and the basalt liquidus at the top of the magma chamber. How-
value can be modified by the disaggregation of the particles consisting the roof rock.

[15] Next, we evaluate \( \delta T \) for the magma chamber (Figure 1b). Here we assume that the magma chamber roof is at a depth of 25 km, a similar depth considered by Ahern et al. [1981]. \( \delta T \) is the temperature increase needed for the roof rock to become partially molten and to disaggre-
agreement, we used \( T_{\text{liq}} \approx 1200^\circ \text{C} \) at \( \approx 10^8 \text{Pa} \) [Hess, 1989], we estimate \( T_{\text{liq}} \approx 1280^\circ \text{C} \) at

\[
\delta T = T_{\text{liq}} - T_{\text{host}},
\]

where \( T_{\text{liq}} \) is the liquidus temperature of the basalt, \( T_{\text{host}} \) is the host rock temperature. Using \( T_{\text{liq}} \approx 1200^\circ \text{C} \) at \( \approx 10^8 \text{Pa} \) [Hess, 1989], we estimate \( T_{\text{liq}} \approx 1280^\circ \text{C} \) at
5. Characteristic Velocity Scales

[16] Dynamically important velocities in our experiments are the erosion velocity \( V_e \) which controls the particle supply from the top, the Stokes settling velocity of the particles \( V_s \), and the thermal convection velocity \( V_c \) which controls the particle entrainment. Larger values of \( V_e \) and \( V_c \) and smaller values of \( V_s \) are preferable for particles to be suspended. Erosion occurs only after the wax between the particles melts, so that the particles can disaggregate and settle. Accordingly, \( V_e \) is rate-limited by the melting velocity \( V_m \). These velocities are indicated in Figure 1c and were measured or estimated using the method described below. A comparison of the velocity scales in the experiments and in the magma chamber are shown in Figures 2a and 10, respectively. Erosion velocities were measured as follows. We select two images before and after the erosion, and measure the eroded area and divide it by the time difference \( t \), to obtain \( V_e \). For the experiments in the rhythmic and transitional cases, we used the time span of the first erosion cycle for \( t \). For the experiments in the continuous case, we used the time span between the start and end of the erosion.

[17] \( V_m \) can be estimated from

\[
V_m = \frac{q}{\rho_w L(1 - \phi_0) + (\rho_p C_p \phi_0 + \rho_w C_w (1 - \phi_0)) \Delta T},
\]
Figure 3. Same as Figure 2c but for a continuous case ($d = 725 \, \mu m$, run 1: Movie 1) which was also shown in Figure 2a3. Colors in the melt represent the temperature field ($red \simeq 47.5^\circ C$, $green \simeq 48.5^\circ C$, and $purple \simeq 54.0^\circ C$). Vertical temperature profile was measured at the right end of the cell. 7250 s: Thermal convection with two downwellings (arrows) and three upwellings. 7500 s: Convection pattern is disrupted by the settling particles but the convective temperature profile persists. 8100 s: Cellular thermal convection reappears with three downwellings (arrows) and two upwellings. Temperature of the melt becomes lower which is evident from the reddish color indicating the temperature field within the melt. 9000 s: As the melt cools, only the upwellings (arrow) are visualized. 11100 s: Melting erosion gradually slows down and finally stops.

Figure 4. Same as Figure 2c but for different particle size ($d = 77 \, \mu m$, run 8: Movie 3), which was also shown in Figure 2a2. Vertical temperature profile was measured at the left end of the cell. 7512 s: Thermal convection with two downwellings (arrows) and three upwellings. 8402 s: Melt layer becomes opaque by suspended particles. 8702 s: Melting erosion and particle settling stops. A two-layered structure forms, the settling front descends downward, and the upper thermal boundary becomes poorly defined. 9002 s: Cold plume appears. 9302 s: The whole suspension layer is mixed.
where \( k \) is the thermal conductivity. Substituting equation (8) into equation (7), and using equation (1), we obtain

\[
V_m = k \left( \frac{ag}{k^2 Ra_c} \frac{1}{1} \Delta T \right)^{1/3} \left[ \rho_L L (1 - \phi_0) + \rho_w C_p \phi_0 + \rho_w C_w (1 - \phi_0) \right]^{1/3} (\delta T)^{1/3},
\]

which is also an upper estimate for \( V_m \). Using the parameters of our experiments, the melting velocity becomes \( V_m = 1.7 \times 10^{-2} \) (m s\(^{-1}\)), which is shown in Figure 2a. Equation (8) is an upper estimate because of two reasons. First, the lateral heat loss to the surrounding area is neglected. Second, as we show in our experiments, particles form a sediment layer at the bottom and can also form a stable density stratification in the melt, both of which suppress the vertical heat transfer. A lower estimate of \( q \) is when the heat transfer is by conduction alone; \( q = k \Delta T / H \). In our experiments, the lower estimate is smaller than the upper estimate by a factor of 13.

In the experiments, \( q \) is supplied by the heater whereas in the magma chamber, it is supplied by the latent heat released from solidification at the bottom and secular cooling.

[18] Stokes settling velocity \( V_s \) and convection velocity \( V_c \) ([Kraichnan, 1962] are estimated from

\[
V_s = \frac{\Delta g d^2}{18 \eta},
\]

\[
V_c = 2 \left( \frac{k}{H} \right) Ra^{1/3} = 2 \left( \frac{ag \Delta T \rho^3}{\nu} \right)^{1/3},
\]

where \( \Delta \rho \) is the particle-melt density difference, \( d \) is the particle diameter, and \( \eta \) is the dynamic viscosity of the melt. For our experimental parameters, from equation (11), we obtain \( V_c = 6.5 \times 10^{-4} \) m s\(^{-1}\), which is shown in Figure 2a. On the other hand, the measured \( V_c \) is \( V_c \approx 1 \times 10^{-3} \) m s\(^{-1}\), which agree with the estimate within a factor of 2. Here we also remark that \( V_c \) is the settling velocity of the particle neglecting the drag from other particles and the wall.

6. Results

6.1. One Particle Size Cases

[19] First, we show the results of experiments with one particle size. From experiments with 11 different particle diameters (Figure 2a, Table 2), we found that for \( d > 100 \) \( \mu \)m, melting erosion and particle settling occur continuously, whereas for \( d < 100 \) \( \mu \)m, they occur rhythmically. For each experiment, we measured erosion velocity \( V_e \) using the method described in section 5, which are plotted in Figure 2a. As described in section 5, \( V_e \) is rate-limited by the melting velocity, and from Figure 2a, we can indeed confirm that \( V_e < V_m \).

[20] Figure 2a3, Figure 3, and Movie 1 show a continuous case with a large particle size \( d = 725 \) \( \mu \)m. Here the particles settle separately one by one without being entrained by the convection, and the erosion proceeds continuously. At the beginning of particle settling, cellular thermal convection is once destroyed but soon the cells reappear and remain thereafter. Vertical temperature profile in the melt (Figure 3) is characterized by thermal boundary layers at the top and the bottom which persist throughout the particle settling.

[21] Figures 2a1, 2a2, 2b, 2c, 4, and Movies 3–5 show examples of rhythmic case for three different small particle
Table 3. A List of Experiments With Two Different Particle Sizes

<table>
<thead>
<tr>
<th>Run</th>
<th>(d_{\text{sm}} - d_{\text{la}}) ((\mu)m)</th>
<th>(V_{\text{sm}} : V_{\text{la}})</th>
<th>(h) (mm)</th>
<th>(\phi_0)</th>
<th>Two-Layers</th>
<th>Case</th>
<th>Rhythmic Layers</th>
<th>Figure, Movie</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>20–153</td>
<td>1 : 1</td>
<td>41.3</td>
<td>0.49</td>
<td>Rhythmic</td>
<td>Rhythmic</td>
<td>Figures 6a, 6b, 7a, 8b, Movie 6</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>20–153</td>
<td>1 : 1</td>
<td>43.9</td>
<td>0.46</td>
<td>Rhythmic</td>
<td>Rhythmic</td>
<td>Figure 8a, Movie 7</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>77–153</td>
<td>1 : 1</td>
<td>37.9</td>
<td>0.53</td>
<td>Rhythmic</td>
<td>Rhythmic</td>
<td>Figure 7c, Movie 9</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>77–153</td>
<td>1 : 1</td>
<td>37.0</td>
<td>0.54</td>
<td>Rhythmic</td>
<td>Rhythmic</td>
<td>Figure 7b, Movie 10</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>77–153</td>
<td>2 : 1</td>
<td>37.7</td>
<td>0.51</td>
<td>Rhythmic</td>
<td>Rhythmic</td>
<td>Figure 8c, Movie 8</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>153–513</td>
<td>1 : 2</td>
<td>34.2</td>
<td>0.56</td>
<td>Continuous</td>
<td>Rhythmic</td>
<td>Figure 2b</td>
<td>22</td>
</tr>
<tr>
<td>23</td>
<td>11–77</td>
<td>1 : 1</td>
<td>37.8</td>
<td>0.51</td>
<td>Rhythmic</td>
<td>Rhythmic</td>
<td>Figure 2c</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Av.(std.) - - 38.4 (2.9) 0.52 (0.03)</td>
<td></td>
</tr>
</tbody>
</table>

\(a\) sm: small, la: large, and \(V\): volume. 11 and 153 \(\mu\)m particles are blue. All other particles are red.  
\(^b\) Localized size grading is evident.

sizes (\(d = 35, 54, \) and 77 \(\mu\)m). Here we describe the representative case \(d = 54 \mu\)m in detail (Figures 2b, 2c, and Movie 4). For this case, as particles settle, they organize themselves into thin granular plumes. Some of the particles which form these plumes are entrained by the convection and are suspended in the melt. As time proceeds, the melt becomes opaque indicating an increase of particle concentration in the melt (Figure 2c, 6300 s). The particle concentration increases toward the bottom to form a sediment where the heat is being transported by conduction alone (Supporting Information Discussion 3.2). Thermal convection becomes sluggish, the erosion of the particles + wax layer becomes slower (see red circles in Figure 2b), and finally the erosion stops. The melt layer then separates into two layers, an upper clear layer and a lower suspension layer (Figure 2c, 9300 s), and the interface descends downward with time. Eventually, a cellular thermal convection develops within the lower layer. Finally, hot plumes ascending from the suspension layer cause an overturn, and the erosion resumes (Figure 2c, 10200 s). These features resemble those observed in layered convection experiments using miscible fluids (see Davaille and Limare [2007] for a review). After the first cycle, when the melting erosion stopped, wax once melted solidified again at the top of the clear layer (arrows in Figure 2b), indicating a strong suppression of vertical heat transfer. The vertical temperature profiles (Figure 2c) also indicate such a temporal change. At 6300 s, the upper and lower thermal boundary layers are evident, indicating that the whole melt layer is convecting. When a two-layered structure forms (9300 s), the upper boundary layer disappears.

[22] Figure 2a1 and Movie 5 show the results for \(d = 35 \mu\)m, and Figures 2a2, 4, and Movie 3 show the results for \(d = 77 \mu\)m, both of which are two other examples of

![Figure 6](image-url). Experimental results for two particle size cases with equal volumes. (a) Case for \(d = 77 \mu\)m (red) and 153 \(\mu\)m (blue) particle mixture (run 19; Movie 6), which by themselves result in rhythmic and continuous erosion, respectively. The images are 10 mm wide sections at the center of the cell and are aligned at intervals of 300 s. (b) Details of the experiment in Figure 6a showing the generation of a rhythmic layering.
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Figure 7. (a) Close up of the sediment layer which formed from an experiment with equal volumes of 77 μm (red) and 153 μm (blue) particles, in which the red particles are smaller than the critical particle size \(d_c\) (run 19: Movie 6, same experiment which was shown in Figure 6). The numbers adjacent to the layers correspond to the erosion cycle numbers. (b) Sediment layer for equal volumes of 11 μm (blue) and 77 μm (red) particles (run 24: Movie 10). Both particle sizes are smaller than \(d_c\) and rhythmic layering forms. (c) Sediment layer for equal volumes of 153 μm (blue) and 513 μm (red) particles (run 23: Movie 9) where both particle sizes are larger than \(d_c\). Sizes of both particles lead to continuous erosion, and rhythmic layering do not form. However, localized size grading is evident.

rhythmic cases. The basic features are the same as that for the 54 μm case. However, for the 77 μm case, the overturn is triggered by the cold plumes descending from the upper clear layer (Figures 2a2 and 4, 9002 s).

[25] Rhythmic case is characterized by a temporal formation of a two-layered structure, and we use this feature to classify the experiments summarized in Table 2. For \(d = 108\) μm (Figure 5, Movie 2), the interface of the two-layers has a limited lateral extent (Figure 5, 8100 s). For this case, we classified the experiment as transitional.

6.2. Two Particle Size Cases

[24] Next, we consider whether rhythmic roof melting can generate a rhythmic layering. Here we use two particles of different diameters and ratios to model the size variation of the mineral particles consisting the roof rock (Table 3). Figures 6a, 6b, 7a, and Movie 6 show the case where the upper solid wax layer contains a mixture of equal volumes of small (red) particles \((d = 77\) μm) and large (blue) particles (153 μm); these particles lead to rhythmic and continuous erosion, respectively. For this case, only the small particles are suspended in the melt and cause the rhythmic erosion. When erosion occurs, both particles settle, whereas when the erosion stops, only the suspended small particles settle. As a result, a set of layers form during each erosion cycle, consisting of a layer of small (red) particles overlying a layer dominated by large (blue) particles. During this experiment, each of the lowermost four layers in Figure 7a was formed by particle settling during the initial four erosion cycles. Layers above these have laterally variable thicknesses, which formed during localized erosional events. This localization arises from the weakening of convection as bottom sediment layer thickens.

[25] Figure 7b and Movie 10 show a case in which both particle sizes \((d = 11\) and 77 μm) are smaller than the critical size \((\sim 100\) μm). For this case, a two-layered structure forms in the melt and a rhythmic erosion occurs. When the erosion stops, the settling is dominated by smaller (blue) particles. As a result, a rhythmic layering forms for each cycle, in which a layer of small (blue) particles overlies a layer dominated by large (red) particles.

[26] In comparison, Figure 7c and Movie 9 show a case in which both of the two particle sizes \((d = 153\) and 513 μm) are larger than the critical size \((\sim 100\) μm). In this case, the particles settled continuously and a two-layered structure did not form. However, we find that the sediment appears size graded compared to the roof rock. From studying the movie and the images, we find that the particle concentration within the melt fluctuates slowly with time, indicating a similar fluctuation of the erosion velocity and hence the particle supply from the top. This suggests that the verti-
cal heat transfer is suppressed as the particle concentration in the melt increases, but the suppression is insufficient to completely stop the erosion and only temporarily reduces the erosion velocity. During this fluctuation, the particles become size graded. However, the size-graded layers are laterally irregular compared to those which formed under rhythmic erosion. Accordingly, we do not classify these layers as a rhythmic layering.

Figure 8 and Movies 6–8 compare the results of three experiments in which the volumetric ratios of the two particle sizes are varied. Here the particles consist of a mixture of particles of two different sizes, \( d = 77 \mu m \) (red) and \( d = 153 \mu m \) (blue), such that one of them is small enough so that rhythmic erosion occurs. Our experiments show that even if only 1/3 of the total volume of the particles consists of the smaller (red) particles, rhythmic erosion occurs (Figure 8c, Movie 8). We note that for this case, there are more layers than the number of erosion cycles, which is different from the cases shown in Figures 8a and 8b. As the volumetric fraction of the larger particles increases, there is a tendency for the location of the roof melting to become more localized. This location moves laterally within one erosion cycle, and as a result, multiple layers form within one cycle.

7. Discussion

7.1. A Criterion for Rhythmic Erosion

Figure 9a is a schematic diagram showing the rhythmic erosion and particle settling. Our experiments indicate that when a suspension becomes sufficiently dense, a stable stratification forms and erosion stops. A condition for a stable stratification to form can be expressed as

\[
B = \frac{\Delta \rho \phi}{\rho \alpha \Delta T} > B_c,
\]

where \( B \) is the Buoyancy number and \( B_c \) is its critical value. Here \( \Delta \rho \) is the particle-melt density difference, \( \phi \) is the particle volumetric fraction in the suspension, \( \rho \) is the melt density, \( \alpha \) is the thermal expansivity, and \( \Delta T \) is the vertical temperature difference in the melt layer. \( B \) has been used previously to evaluate the stability of the stratification in layered thermal convection [Richter and McKenzie, 1981; Olson and Kincaid, 1991; Davaille, 1999b; Namiki, 2003; Le Bars and Davaille, 2004; Davaille and Limare, 2007], which showed that \( B_c \) is of the order of 0.1–1. \( B \) has also been used to classify flows in thermal convection containing particles [Höink et al., 2005, 2006].

In our experiments, as particles settle, \( \phi \) in the melt increases with time. Since smaller particles settle slower, they form denser suspensions (large \( \phi \)), which results in larger value of \( B \). This qualitatively explains the formation of a stable stratification below a critical particle size. Here we apply the Buoyancy number criterion and estimate \( B_c \) for our experiments.

We can estimate the maximum value \( \phi_{max} \) for a given particle size as follows. The mass balance of the particle content within the melt can be expressed as

\[
\frac{d}{dt}(\phi(t)H) = \phi_0 V_e - \phi(t)V_s,
\]

where \( \phi(t) \) is the particle volumetric fraction within the melt layer with a thickness \( H \). The first term on the right-hand side of equation (13) represents the particle flux entering into the melt layer by erosion, and the second term is the particle flux exiting from the melt layer by sedimentation.

At \( t = 0 \) (i.e., at the onset of particle settling), \( \phi(t) = 0 \). A solution of equation (13) is given by

\[
\phi(t) = \frac{V_s \phi_0}{V_e} \left(1 - \exp \left(\frac{-t}{\tau}\right)\right),
\]

where \( \tau = \frac{H}{V_e} \) is the characteristic time scale required for the particle content within the melt layer to increase to a steady state value.

We note that when solving equation (13) to obtain equation (14), we assumed that \( H, V_e, \) and \( V_s \) are constants, i.e., do not depend on time during the period of erosion. Here we evaluate their constancy. For the case shown in Figures 2b and 2c, for example, during the first erosion period (5660–8470 (s)), \( H \) is constant to within \( \pm 7\% \) whereas \( V_e \) decreases with time by about a factor of 2. As particle content \( \phi \) rises, interparticle drag causes \( V_e \) to decrease. This hindered settling velocity scales as \( V_e \propto (1-\phi)^{1/3} \) [Davis and Acrivos, 1985]. Substituting \( \phi = \phi_{max} \), we obtain a characteristic time of \( \tau \) (see below), we find that \( V_e \) decreases by 8%.

As erosion proceeds, \( \phi(t) \) increases with time, until the erosion flux balances the sedimentation flux, and \( \phi(t) \) approaches a steady state maximum value of

\[
\phi_{max} = \left(\frac{V_s}{V_e}\right) \phi_0.
\]

Figure 2a shows that \( V_e \) does not depend on the particle size, which we interpreted as being rate-limited by melting velocity. Accordingly, we have an approximate relation of \( \phi_{max} \propto V_e^{-1} \propto 1/d^2 \), indicating that \( \phi_{max} \) increases as particle diameter \( d \) decreases. A steady state is achieved only when the erosion is continuous. In the rhythmic case, the erosion stops before the steady state is achieved. We can consider that the particle size of the transitional case is at the critical value \( \phi_{max} = 0.9 \) (see below), \( B \) becomes large enough and \( B \) is of the order of 0.1–1. \( B \) has also been used to classify flows in thermal convection containing particles [Höink et al., 2005, 2006].

Here we estimate \( \phi_0 \) for our experiments. Substituting \( V_e = 3.5 \times 10^{-4} \text{m s}^{-1} \), \( V_s = 1.2 \times 10^{-4} \text{m s}^{-1} \), \( \phi_0 = 0.565 \), which correspond to the values of the transitional cases (Table 2), into equation (15), we obtain \( \phi_0 = 0.017 \). Substituting \( \phi = \phi_{max} \), together with the physical properties of molten wax and glass beads (Tables S1 and S3) into equation (12), we obtain \( B_c = 0.9 \) which is \( \sim 1.0 \). A value close to unity is consistent with those obtained from previous works on layered convection. We also obtain \( \phi_{max} = 0.03 \), which is the ratio of the two velocity scales at the transition (Figure 2a). In addition, using \( H = 49.8 \text{mm} \), we obtain a characteristic time of \( \tau = 429 \) (s). The measured time between the initiation and cease of the erosion of the first cycle in these two experiments is 1270 ± 509 (s) and agrees with \( \tau \) within a factor of 3.

7.2. Critical Particle Size for Rhythmic Erosion

Here we derive a theoretical expression for the critical particle size below which a rhythmic erosion occurs.
Figure 9. A model of a rhythmic erosion and its application to the magma chamber. (a) Schematic diagram of a rhythmic erosion with large (blue) and small (red) particles. (b) A summary of the experiments and a theoretical line for a critical particle size calculated from equation (18) with $\phi_0 = 0.565$. Black and red broken lines are calculated using $B_c = 0.8$ and $B_c = 1.2$, respectively. (c) Conditions for rhythmic and continuous erosion to occur in a basaltic magma chamber which intrudes into a more mafic roof rock. Here we use $B_c = 1.0$, $V_e = 0.2 V_m$. Solid and broken lines are calculated using magma viscosities of $\mu = 100$ and $10$ Pa s, respectively. Red and black lines are calculated using $\phi_0 = 0.5$ and $0.7$, respectively, latter value is when the contact between the grains diminishes [Scott and Kohlstedt, 2006].

From Figure 2a, we may assume the following relation between $V_e$ (measured) and $V_m$ (an upper estimate given by equation (9));

$$V_e = c V_m,$$  \hspace{1cm} (16)

where $c$ is an empirical proportional constant. Since we are interested in the critical condition, we use $V_e$ of the transitional case to obtain $c = 0.2$. We remark that Shibano et al. [2012] conducted experiments for the case in which the particles consisting the roof disaggregate without melting. For such case, different from the experiments in this work, the erosion velocity increases with particle size.

Using equation (16), we can rewrite equation (15) as

$$\phi_{\text{max}} = \left( \frac{c V_m}{V_e} \right) \phi_0.$$  \hspace{1cm} (17)

From $B = B_c$, we can solve for the critical particle size ($d_c$) below which rhythmic erosion occurs, as follows. We substitute equations (9) and (10) into equation (17), and then substitute $\phi = \phi_{\text{max}}$ into equation (12). Solving for $d$, we obtain an expression for the critical particle size $d_c$ as

$$d_c = \Delta T^{1/6} \left( \frac{1}{\rho a} \right)^{1/2} \left( \frac{c^3 k^{1/3} \alpha g}{\gamma V R a c_0} \right)^{1/6} \left[ \rho_c L (1 - \phi_0) + \rho_c C_w (1 - \phi_0) \right]$$

$$+ \rho_c C_w \phi_0 \delta T^{1/2} \left( \frac{g}{18 \eta} \right)^{1/2} \phi_0^{1/2} B_c^{-1/2} \alpha c^{1/2} \Delta T^{1/6} \eta^{1/3} B_c^{1/2}.$$  \hspace{1cm} (18)

Note that equation (18) shows that $d_c$ does not depend on the density difference $\Delta \rho$. This is because $B$ does not depend on $\Delta \rho$, which arises from $B \propto \Delta \rho \phi$ (equation (12)), and $\phi$ depending on $\Delta \rho$ as $\phi \propto V_e^{-1/2} \Delta \rho^{-1}$.

We use equation (18) to calculate the line in Figure 9b, separating the continuous and rhythmic cases for values $B_c = 0.8$ and 1.2. Figure 9b confirms that $B_c \simeq 1.0$ separates the two cases, a result which we obtained in section 7.1.

7.3. An Estimate of Minimum Erosion Thickness Scale Per Cycle

We can also estimate the minimum thickness of the particles + wax layer, $\delta$ that needs to be eroded to form a suspension layer with a particle volumetric fraction of $\phi_c$.

From mass balance,

$$\delta \phi_0 = H \phi_c.$$  \hspace{1cm} (19)

Since $\phi_c$ can be expressed as

$$\phi_c = \left( \frac{\rho a \Delta T}{\Delta \rho} \right) B_c,$$  \hspace{1cm} (20)

from equations (19) and (20), we obtain

$$\delta = H \left( \frac{\rho a \Delta T}{\Delta \rho} \right) \left( \frac{B_c}{\phi_0} \right).$$  \hspace{1cm} (21)
Equation (21) relates the erosion thickness $\delta$ to the experimental parameters with $B_\ell = 1.0$.

[30] Not all particles which are eroded form the suspension and some settle out. A minimum estimate of the thickness $\delta'$ which is eroded but settle out can be estimated as follows. $\delta'$ can be estimated from the thickness of the sediment which has settled during the characteristic time $\tau$ as

$$\delta' = \frac{1}{\phi_0} \int_0^\tau \phi(t) V_s dt = \frac{V_s H}{V_c e}.$$  \hspace{1cm} (22)

[40] For our experiments, substituting $\phi_0 = 0.565$ and $H = 49.8$ mm, which are the average of the two data of the transitional cases (runs 6 and 7), into equation (21), we obtain $\delta = 1.7$ mm. Substituting $V_c = 3.5 \times 10^{-4}$ m s$^{-1}$ and $V_s = 1.2 \times 10^{-4}$ m s$^{-1}$ into equation (22), we obtain $\delta' = 0.55$ mm. A minimum estimate of the eroded thickness therefore becomes $\delta + \delta' \approx 2.3$ mm. In comparison, the measured average thickness of the eroded particles + wax layer during the first erosion cycle for these two experiments is $4.2 \pm 0.8$ mm and agrees with the estimate within a factor of 2.

7.4. Comparison With Previous Works

[41] First, we compare with the results of layered convection experiments using miscible fluids. Davaille, [1999a, 1999b] conducted experiments using a cellulose solution ($Pr \sim 10^2 - 10^3$) and studied the effects of the viscosity ratio of the two layers. Here $Ra$ was defined for each layer, and the larger $Ra$ of the two layers was in the range of $10^6 - 2 \times 10^7$. For a viscosity ratio of ~ 1 which corresponds to those of our experiments, they showed that for $B > 1$, a stable stratification forms, whereas for $B < 0.4$, domes form which eventually break up and mix. Thisdoming, breaking up and the final mixing of the two layers, is similar to the processes shown in Figures 2a1, 2c, and 4. Davaille [1999b] also showed that some experiments start in the stratified regime and end in the doming regime, a feature corresponding to the decrease of buoyancy number with time. This feature is also common to what we observe in our experiments.

[42] Second, we review the experiments by Koyaguchi et al. [1993] in terms of buoyancy number. They conducted experiments using water ($Pr = 7$) containing silicon carbide particles ($\Delta \rho = 2217$ kg m$^{-3}$) which was heated from below resulting in $Ra = 2 \times 10^8$. They showed that when the particle weight fraction was $c_w > 0.3$ wt%, thermal convection was suppressed and the convection became confined to the suspension layer and eventually an overturn occurred. However, for $c_w = 0.5$ wt%, the overturn did not occur. Particle concentrations of $c_w = 0.3$ wt% and $c_w = 0.5$ wt% correspond to particle volumetric fraction of $\phi = 9.3 \times 10^{-4}$ and $1.6 \times 10^{-3}$, respectively. Using $\alpha = 2.1 \times 10^{-4} \text{K}^{-1}$ for water and $\Delta T = 24^\circ \text{C}$ in their experiments, the buoyancy numbers of the above two cases become $B = 0.4$ and $B = 0.7$, respectively. By taking an intermediate value, this suggests a critical value of $B_{\ell} \sim 0.55$ for their experiments. This value is also close to the critical buoyancy number of $B_{\ell} \approx 1.0$ obtained for our experiments.

[43] We note that although the Rayleigh numbers ($Ra$) of the above works differ from the $Ra$ of our experiments by up to 3 orders of magnitude, the value of the critical buoyancy number remains unchanged and is of the order of 0.1–1.

8. Implications to Magma Chamber

8.1. Can Rhythmic Erosion Occur in a Magma Chamber?

[44] Here we apply the $B > 1.0$ criterion for rhythmic erosion to occur, to a situation in which a superheated basaltic magma melts a more mafic roof rock. This situation most closely resembles our experiments because the intruding magma does not contain crystals, and the density and viscosity of the magma and the roof rock melt are similar. Note that similar phenomena can be expected to occur even when the magma intrudes into the roof rock having a comparable or a silicic composition (see section 1). We similarly use equation (18) to evaluate the critical particle size below which rhythmic erosion occurs which is shown in Figure 9c. Here we considered a vertical temperature difference of $1 \leq \Delta T \leq 100^\circ \text{C}$, which corresponds to a magma chamber thickness of $400 \text{ m} \leq H \leq 40 \text{ km}$ if we use equation (5) and covers the typical thickness scale of large mafic intrusions [e.g., Hess, 1989; Philpotts and Ague, 2009]. The value of $c$ is in the range of $0 < c \leq 1$. Here we assumed $c = 0.2$ obtained from the experiments. Cases for different magma viscosities $\eta$ and volumetric packing fraction of the particles consisting the roof rock $\phi_0$ are also shown.

[45] Figure 9c shows that for a given grain size, rhythmic erosion is more likely to occur in a magma chamber with a large temperature difference (i.e., a thicker magma chamber) and a larger magma viscosity. This is because the roof melting velocity becomes faster, and the particle settling velocity becomes slower, respectively. The figure shows that for $\Delta T = 10^\circ \text{C}$, which corresponds to $H = 4000 \text{ m}$ assuming equation (5), and a magma viscosity of $\eta = 100 \text{ Pa s}$, grain size of $< 0.6 \text{ mm}$ leads to allow rhythmic erosion. This size range overlaps the typical grain size range of host rocks (~0.1–10 mm) [Philpotts and Ague, 2009], suggesting that rhythmic roof melting is a viable process to generate the rhythmic layering. This also implies that under these conditions, magma may ascend intermittently.

[46] Next, we estimate other characteristic scales related to rhythmic erosion. The critical particle volumetric fraction $\phi_0$ can be estimated using equation (20) as

$$\phi_0 = \left( \frac{\rho a \Delta T}{\Delta \rho} \right) B_e = 2 \times 10^{-3} \left( \frac{\Delta T (\circ \text{C})}{10^\circ \text{C}} \right).$$ \hspace{1cm} (23)

This indicates that a fairly dilute suspension is capable of forming a stable stratification, and hence resulting in rhythmic erosion. The corresponding minimum erosion thickness per cycle can be estimated using equations (21)–(22). We assume $\phi_0 = 0.5$ and obtain

$$\delta = 16 \text{ (m)} \left( \frac{H (\text{m})}{4000} \right) \left( \frac{\Delta T (\circ \text{C})}{10} \right).$$ \hspace{1cm} (24)

Using $d_e = 0.6 \text{ mm}$, from equations (5) and (22), we obtain,

$$\delta' = 5 \text{ (m)} \left( \frac{H (\text{m})}{4000} \right) \left( \frac{\Delta T (\circ \text{C})}{10} \right)^{4/3}.$$ \hspace{1cm} (25)

As a result, from equations (24) and (25), we obtain the minimum erosion thickness per cycle as $\delta + \delta' = 21 \text{ m}$ for a magma chamber size of $H = 4000 \text{ m}$. We can also estimate
the characteristic time scale in equation (14) using a critical particle size of $d_c = 0.6$ mm and obtain $\tau = H/V_s \approx 100$ years.

Our experiments indicate that the thickness of the eroded roof rock per cycle is comparable to the thickness of the sediment formed per cycle. Accordingly, $\delta + \delta_c$ can also be considered as an estimate of the thickness of the rhythmic layer per cycle. However, the typical thickness of the actual rhythmic layering observed in the outcrops is of the order of $\approx 1$ m or less [e.g., Naslund and McBirney, 1996]. Our estimate of a layer thickness of $\approx 21$ m for a magma chamber size of $H = 4000$ m is quite large compared to those of the outcrops. Formation of multiple layers per each erosion cycle may partially explain this difference. Multiple layers formed in our experiments for the case in which the volumetric fraction of the larger of the two particle sizes is 0.67 (Figure 8c). In addition, particles can be simultaneously supplied from crystallization in the cooling magma, which also aids in forming multiple layers per erosion cycle (see section 8.2). We also remark that from combining equation (5) and (24), we obtain $\delta \propto H^2$, indicating that a smaller magma chamber can result in a smaller layer thickness.

Unless the magma is being reinjected, the magma chamber loses its heat to the surrounding host rock and will finally solidify. Accordingly, for a rhythmic erosion to occur, the characteristic time scale $\tau$ should be much shorter than the time scale for magma chamber to cool by heat conduction, which can be expressed as

$$\tau = \frac{H}{V_s} \ll \frac{H^2}{\kappa}.$$  \hspace{1cm} (26)

We can estimate the minimum magma chamber thickness $H$ which satisfies equation (26) as

$$H \gg 8 \text{ m} \left( \frac{\eta}{100 \text{ Pa s}} \right) \left( \frac{0.1 \text{ mm}}{d} \right)^2.$$  \hspace{1cm} (27)

Here we used the grain size $d = 0.1$ mm which is around the upper limit size for the rhythmic case to occur (Figure 9c). This estimate suggests that when the basaltic magma chamber thickness exceeds of the order of 10 m, cooling of the magma chamber is slow enough such that rhythmic erosion may occur, provided that $d < d_c$.

Finally, in Figure 10, we compare the characteristic velocity scales in a basaltic magma chamber using the same parameter values. Comparing with Figure 2a, we find that relative magnitude relations of the velocities are similar to those of the experiments. We also note that in the magma chamber, $V_s > V_c$, suggesting that the mineral particles disaggregate and settle as soon as it becomes partially molten, which is also similar to the experiments.

8.2. Implications for the Origin of Rhythmic Layering

Several models for the origin of rhythmic layering have been proposed [see Naslund and McBirney [1996] for a review]. A group of models considers intermittent convection, and our experiments can also be classified into this group. For example, Brandeis and Jaupart [1987] showed that in a magma chamber which is cooled from top and from below, cold plumes form intermittently. They proposed that these cold plumes can bear crystals and deposit the crystals at certain time intervals. Sparks et al. [1993] considered a thermally convecting magma chamber in which the crystal content increases with time as it cools. Based on the experiments by Koyaguchi et al. [1993], they proposed that when the crystal content exceeds a critical value, the thermal convection is suppressed and particles settle, after which thermal convection resumes and the cycle repeats itself.

Our laboratory model for generating a rhythmic layering is similar to the conceptual model by Sparks et al. [1993]. The difference is that Sparks et al. [1993] consider particles formed from crystallization in a cooling magma, whereas we consider particles which have disaggregated from roof rock melting. Our experiments indicate that rhythmic layering can be generated from roof rock melting alone. However, roof melting and crystallization of the magma can occur simultaneously (Figure 1a). In such case, particles formed from crystallization can aid in forming a stable stratification. In addition, cold settling particles from the roof can induce crystallization in the magma [Huppert and Sparks, 1988a], and therefore these two mechanisms can become coupled. Accordingly, we may view our model and that by Sparks et al. [1993] as two simple end-member cases, which differ in the mechanism of particle supply. Perhaps, similar phenomena which we observed in our experiments occur even when the particles are supplied by crystallization.

Our model may be relevant for the case in which the vertical compositional variation cannot be explained by assimilation of the roof rock and fractional crystallization alone. Particle settling arising from roof melting efficiently concentrates refractory minerals as sedimentary cumulates, and may be associated with the formation of olivine-rich rocks such as dunite. Another feature of our model is that the total resulting thickness of the sediments may become comparable to, or even exceed the initial size of the magma chamber itself.
8.3. Effects Not Considered in the Experiments

[55] Our experiments focus on the consequences of roof melting and the subsequent particle settling. Here we consider the effects which are not modeled in our experiments.

[54] First, our experiments do not consider the cooling and crystallization of the magma chamber. With cooling, the vertical temperature difference of the magma chamber \( \Delta T \) will decrease, whereas the magma viscosity \( \eta \) will increase. Critical particle size \( d_c \) depends on these parameters as \( d_c \propto \Delta T^{1/6} \eta^{1/3} \) (equation (18)), which is also shown graphically in Figure 9(c). The decrease of \( \Delta T \) causes \( d_c \) to become smaller, whereas the increase of \( \eta \) causes \( d_c \) to become larger. This implies that as the magma chamber cools, rhythmic erosion may change to continuous erosion or vice versa. As cooling proceeds, eventually heat is being transferred by conduction alone after which erosion stops and the solidification of the whole magma chamber occurs. Processes related to cooling such as density currents caused by wall crystallization and formation of layers within the magma chamber by double diffusion [e.g., Turner, 1980] are also not modeled in our experiments.

[55] Second, the experiments assumed simple physical properties for the magma, roof melt, and particles. We assumed that the physical properties of the intruding magma and the roof melt are the same. The density and viscosity contrasts between the magma and the melt formed from roof melting will affect the mixing and the stratification of the magma chamber [e.g., Huppert and Sparks, 1988b] and the resulting erosion style. We assumed that the particles are spherical. The actual mineral particles are aspherical and their settling velocities will differ. We remark that we formulated our theoretical model without considering the effect of particle suspension on viscosity, and assuming Newtonian fluids, because particle concentration in the melt is a priori unknown. However, our estimates for particle concentration in the experiment (\( \phi \approx 0.017 \), see section 7.1, equation (15)) and in the magma (\( \phi \approx 0.017 \), see equation (23)) indicate that the viscosity increase is small (< 7%, see Table S1) if we assume that the particles are homogeneously distributed. Therefore, for a superliquidus magma, the deviation seems to be small. However, if the model is applied to a subliquidus magma, the viscosity of the partial melt needs to be used instead of the melt viscosity.

[56] Third, in addition to roof melting, the magma may ascend by viscously deforming the host rock at the same time, i.e., ascent by diapirs. Figure 11 compares the magma ascent velocity by roof melting with diapirc ascent velocity, as a function of magma chamber thickness \( H \). The melting velocity is calculated using equation (9), using \( \phi_0 = 0.5 \), \( \delta T = 580^\circ \text{C} \), and other parameter values are from Tables S1–S3. \( \Delta T \) is related to \( H \) using equation (5). Erosion velocity is calculated using equation (16), with \( c = 0.2 \) obtained from the experiments. Diapirc ascent velocity is calculated from

\[
V_{\text{diapir}} = \frac{\Delta \rho g H}{18 \eta_{\text{host}}}.
\]

where \( \eta_{\text{host}} \) is the viscosity of the host rock. Here we use \( \eta_{\text{host}} = 10^{19} \text{ Pa s} \), which is an estimate for the mantle asthenosphere [Turcotte and Schubert, 2002]. Comparison shows that magma ascent by roof melting overwhelms that by diapirc ascent, unless the magma chamber thickness becomes as large as \( H \approx 10^4\text{ m} \). We note, however, that when the magma chamber cools and convection ceases, the diapirc ascent velocity may overtake the erosion velocity.

9. Conclusions

[57] We have conducted a series of experiments modeling the melting erosion of the roof rock of a thermally convecting magma chamber and the subsequent particle settling from the partially molten roof. We find that when the particle size consisting the partially molten roof rock is smaller than a critical size, melting erosion occurs rhythmically. Erosion stops when the particle concentration within the magma chamber becomes sufficiently high such that stable stratification forms. Erosion resumes by the overturning of the stratified magma chamber as it is heated from below. When there are two particle sizes, in which at least one of them is smaller than the critical size, rhythmic roof melting occurs and a rhythmic layering is generated. We showed that a criterion for a rhythmic erosion to occur can be expressed as \( B > 1.0 \), where \( B \) is the dimensionless Buoyancy number. When we apply the same criterion to a basaltic magma chamber, we find that rhythmic erosion can occur when the grain size of the particles consisting the roof rock is \( < 0.6 \text{ mm} \), assuming a vertical temperature difference of \( \Delta T = 10^6\text{ C} \). This indicates that the particle settling which results from the melting erosion of the roof rock is a possible mechanism for generating the rhythmic layering which is commonly found in fossil magma chambers. It also implies that under this condition, magma will ascend intermittently.

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