Experiments on buoyancy-driven crack around the brittle–ductile transition

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We report the results of laboratory experiments exploring how a buoyancy-driven liquid-filled crack migrates within a viscoelastic medium whose rheology is around the brittle–ductile transition. To model such medium, we use a low concentration agar, which has a small yield stress and a large yield strain (deformation) when it fractures. We find that around the transition, the fluid migrates as a hybrid of a diapir (head) and a dyke (tail). Here the diapir is a bulged crack in which fracturing occurs at its tip and closes at its tail to form a dyke. A small amount of fluid is left along its trail and the fluid decelerates with time. We study how the shape and velocity of a constant volume fluid change as two control parameters are varied; the agar concentration (C) and the density difference \( \Delta \rho \) between the fluid and the agar. Under a fixed \( \Delta \rho \), as C decreases the medium becomes ductile, and the trajectory and shape of the fluid changes from a linearly migrating dyke to a meandering or a bifurcating dyke, and finally to a diapir–dyke hybrid. In this transition, the shape of the crack tip viewed from above, changes from blade-like to a cusped-ellipse. A similar transition is also observed when \( \Delta \rho \) increases under a fixed C, which can be interpreted using a force balance between the buoyancy and the yield stress. Our experiments indicate that cracks around the brittle–ductile transition deviates from those in an elastic medium by several ways, such as the relaxation of the crack bulge, slower deceleration rate, and velocity becoming insensitive to medium rheology. Our experiments suggest that the fluid migrates as a diapir–dyke hybrid around the brittle–ductile transition and that fluid migration of various styles can coexist at the same depth, if they have different buoyancy.

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1. Introduction

Ascent of buoyant magma in the lithosphere is one of the elementary processes of volcanism in terrestrial planets as well as in icy satellites. Two very different end-member styles, namely dykes and diapirs, have been considered as the styles of magma transport (e.g., Parfit and Wilson, 2008). Dykes form by brittle fracture at the crack tip and elastic deformation of the host rock. Elastic response is rapid whereas the viscous flow of magma into the narrow crack tip is slow, and as a consequence, magma viscosity controls the velocity of magma ascent. On the other hand, a diapirc rise of magma occurs by ductile (viscous) deformation of the host rock. Since the viscosity of the host rock is much larger than that of the magma, the host rock viscosity, rather than the magma viscosity, controls the ascent velocity. These end-member cases have been extensively studied using analytical, numerical and experimental methods.

In Earth, the host rock rheology changes from brittle to ductile as the depth increases. In the continental lithosphere, the brittle–ductile transition is considered to occur at a depth of \( \sim 15 \text{ km} \) (Scholz, 2003). This depth is shallower than the typical depth of \( \sim 100 \text{ km} \) at which the magma is being generated (Asimow, 2000). Brittle–ductile transition of rocks is considered to occur gradually over a temperature and pressure range (e.g., Hirth and Tullis, 1994; Kohlstedt et al., 1995). As a consequence it is conceivable that the rising magma gradually experiences a change of the host rock rheology from ductile to brittle. There have been several experiments focusing on the dynamics around the brittle–ductile transition (Gladden and Belmonte, 2007; Hirata, 1998) but these were for the cases in which the gravity was unimportant. There have also been some theoretical studies such as by Rubin (1993) who calculated the shape of the rising magma body in a Maxwell viscoelastic medium. However in this study, the fracture resistance of the host rock was neglected, and the case in which the ductile fracturing occurs remained unexplored. Therefore, our knowledge on how the magma rises in the brittle–ductile transition zone is still quite limited.

In this paper, we report the results of laboratory experiments designed to explore how the buoyancy-driven liquid-filled cracks migrate around the brittle–ductile transition. We use an agar of a low concentration to model such medium rheology. Agar is similar to gelatin which has been used in many experiments on buoyant or pressurized cracks to model the brittle lithosphere e.g., (Heimpel and Olson, 1994; Ito and Martel, 2002; Menand and Tait, 2002; Rivalta and Dahm, 2006; Taisne and Tait, 2009; Takada, 1990; Watanabe et al., 2002). We use a low concentration agar which has a shear modulus that is up to 3 orders of magnitude smaller than those of gelatin used in previous experiments. Such a soft agar fractures under a small yield
stress and a large yield strain. Using such agar, we study how the shape and velocity of a migrating constant volume fluid changes when we vary 2 parameters; the density difference and agar concentration, which allow us to control the buoyancy force and the medium rheology, respectively.

2. Experimental methods

The experimental setup is shown in Fig. 1. A cylindrical acrylic tank contains an agar whose physical properties are described in Section 3.

A fluid with a constant volume of $1 \pm 0.025\text{cm}^3$ is injected in 1–2 s from the center of the upper lid using a syringe with an inner diameter of 0.94 mm. Injection of a constant volume fluid corresponds to the case in which the magma supply ceases after a certain time. Alternatively fluids can be injected at a constant flux, which corresponds to magma supply which is sustained during its ascent. Most of the previous experiments on buoyancy-driven cracks have been done at a constant volume, and for the ease of comparison, we similarly conduct our experiments.

For the fluids, we use aqueous solutions which are miscible with agar such that there is no surface tension between them. Presence of surface tension affects the shape of the injected fluid and the amount of fluid which is being left along its trail. The density is varied by using aqueous solutions of different concentrations. We use distilled water, NaCl solution (4.0, 12.0, and 24.0 wt.%), CsCl solution (7.0, 16.0, 30.0, 40.0, 50.0, and 60.0 wt.%) which have densities in the range of 1005–1792 kg m$^{-3}$ at room temperature. The error of the density difference between the injected fluid and the agar is $\leq 2$ kg m$^{-3}$. The fluids are colored in red using a dye (0.1 wt.%). At 20 °C, the viscosity of NaCl solution increases with solute concentration and is in the range of $(1.002 \text{mPas})$ (Lide, 2005). When the density difference is less than the critical, the crack does not propagate. To clarify the critical condition, 12 experiments at agar concentrations of 0.94 mm. Injection of a constant volume

3. Agar and its rheology

We use an agar (Ina Food Industry, UP-6) with a concentration (C) in the range of 0.06–0.4 wt.%. Agar is similar to gelatin, but solidifies above room temperature. The density of agar increases slightly with concentration and at room temperature it increases as $995–999 \pm 1$ kg m$^{-3}$.

Agar is prepared by completely dissolving the agar powder in a boiling distilled water using a microwave oven, and then diluting it with water to a specified concentration. We allow the agar to cool for at least 24 h, after which we find that there is no systematic increase of yield stress.

The rheology of the agar was measured using Brookfield viscometers (DVII+PRO) with a vane spindle (V-73) attached, a technique which has been used previously for measuring gels (Alderman et al., 1991). We use 3 viscometers, Brookfield LV, RV, and 2HB models, where RV and 2HB models have spring constants that are 10.7 and 170.7 times larger than the LV model such that they can measure stiffer agar. The LV, RV, and 2HB viscometers were used to measure the rheology of agar at $C = 0.06–0.08$, 0.1–0.15, and 0.2–0.4 wt.%, respectively. We insert the vane spindle vertically into the agar and rotate it to obtain the stress–time series data at 10 Hz. For shearing the sample, we use 3 cylindrical containers S, M, and L which have diameters of 30, 44, and 54 mm and heights of 62, 56, and 108 mm respectively.

Fig. 2(a) shows the examples of the stress–time data. If the sample does not deform, the stress increases linearly in time which is indicated by a broken line in Fig. 2(a). When the sample deforms, the slope of the stress–time curve becomes smaller compared to the broken line. We show two examples for agars with different concentrations that are sheared at the same stress loading rate (i.e., using same viscometer and same rotation rate). The slope of the stress–time curve becomes steeper for a stiffer agar. We define the stress $\sigma_y$ needed to fracture the agar by the peak stress (indicated by circles) of the stress–time curve. The figure shows that an agar with a higher concentration is stiffer and has a larger yield stress, followed by a large stress drop. This feature indicates that with increase in C, the style of fracturing gradually changes from ductile to brittle.

In Fig. 2(b), we plot $\sigma_y$ as a function of C for different rotation rates (in rotation per minute, hereafter rpm), and container size. The accuracy of the viscometer is 1% of the maximum torque which can be imposed by the viscometer. Accordingly we only use the results in which $\sigma_y$ is above this threshold and similarly evaluate the errors. The plot shows that $\sigma_y$ increases by 1–3 orders of magnitude as C increases from 0.06 wt.% to 0.4 wt.%. We also note that $\sigma_y$ is rate-dependent. For $C \leq 0.1$ wt.%, $\sigma_y$ is rate-strengthening, whereas for $C \geq 0.15$ wt.%, $\sigma_y$ is rate-weakening. Since $\sigma_y$ is rate-dependent, we choose to present our experimental results on fluid migration as a function of C, rather than $\sigma_y$. The plot also shows that there is no clear dependence of $\sigma_y$ on the size of the sample container used for shearing.

Shear modulus G is calculated using the method described in Alderman et al. (1991) and the results are plotted in Fig. 2(c). G is proportional to the slope of the stress–time series data and we use the maximum slope (see Fig. 2(a)). Here we used N data points ($N \geq 4$) of the stress data covering more than half of the data points in the range of $\sigma_{y\text{min}} < \sigma < \sigma_y$, where $\sigma_{y\text{min}}$ is the minimum stress value, and calculate the slope by the least squares method. The error bars are evaluated from the error of the slope. The plot shows that G increases by about 4 orders of magnitude as C increases from 0.06 wt.% to 0.4 wt.% and that its rate-dependence is small. G of gelatin used in previous experiments on buoyancy-driven cracks have values of around $G = 190–2600$ Pa. Comparing with our results, it seems that agar in the concentration range of $C = 0.3$ wt.% approximately corresponds to those of the gelatin used in the previous experiments.

Using the above results, we can calculate the yield strain $\gamma_y$ (Alderman et al., 1991) as

$$\gamma_y = \frac{\sigma_y}{G}$$

(1)
which we plot in Fig. 2(d). The plot shows that for small C, the yield strain is large ($\gamma_\sigma > 1$). $\gamma_\sigma$ decreases by an order of magnitude with C, a consequence of $G$ being more sensitive to $C$ than $\sigma_y$.

We also calculated the strain rate at the cylindrical surface defined by the spindle using the same data that were used to calculate the shear modulus. For calculation, we use the size of the container assuming that the elastic deformation occurs throughout the sample. We find that the shear rate varies approximately in proportion to the rotation rate, and is of the order of $0.01 \, s^{-1}$ at $0.1 \, rpm$ to $10 \, s^{-1}$ at $100 \, rpm$, regardless of the type of viscometer (spring constants) used. The rotation rate of $10 \, rpm$ corresponds to a shear rate of $1 \, 100 \, rpm$, regardless of the type of viscometer (spring constants) used.

4. Comparison of the rheology of agar, rock and ice

Here we compare the rheology of agar described in the previous section with those of rock and ice.

In the Earth’s lithosphere, the strength of the rock first increases with depth (brittle regime) and then decreases with depth (ductile regime). As a consequence there is a strength maximum at a certain depth (e.g., Kohlstedt et al., 1995). Since agar becomes ductile and softer as its concentration ($C$) decreases, this suggests that we may model the rock rheology below the depth at which the strength becomes maximum by decreasing $C$ (Fig. 2(b)–(c)).

We can make further comparisons by examining the rate-dependence of the yield stress (Fig. 2(b)). This property has also been studied for ice from compression and tension tests at 1 atm and at temperatures of $-50 < T < 0 \, ^\circ C$ (e.g., Schulson, 2001; Xu et al., 2004). At low strain rates, ice is ductile and is rate-strengthening whereas above the critical strain rate of $\gamma = 10^{-4} - 10^{-5} \, s^{-1}$, ice becomes brittle and becomes rate-weakening. Furthermore, the apparent Young’s modulus also changes with strain rate with a maximum near the brittle–ductile transition. A ductile agar at $C \leq 0.1 \, wt.%$ is also rate-strengthening whereas brittle agar at $C \geq 0.12 \, wt.%$ is rate-weakening (Fig. 2(b)), and is similar to ice. Rate-dependence of shear modulus (Fig. 2(c)) is not obvious, though the data do indicate a rate-weakening behavior in the brittle regime. Therefore the low C and high C agars are qualitatively similar to ductile and brittle ice, respectively.
We can also compare the magnitude of the yield strain (Fig. 2(d)). Deformation experiments of rock and ice indicate that yield strain $\gamma_y$, when defined as the strain at the peak stress, is typically $\gamma_y \approx 0.01$. Although some large strain experiments (see Mackwell and Paterson (2002) for a review) do indicate a $\gamma_y$ as large as of the order of $\sim 1$, most values of $\gamma_y$ are smaller than that of agar ($\gamma_y \sim 0.1 - 0.15$). Here we note that $\gamma_y$ is also rate-dependent. $\gamma_y$ of rock salt, for example, decreases with strain rate (Dubey and Gairola, 2005).

Importantly, strain rates of geological phenomena are much smaller than those of the deformation experiments. For example, when the magma body with a length scale of $l$ rises within a host rock with a dynamic viscosity of $\eta$, the strain rate can be estimated as

$$\dot{\gamma} = \frac{\Delta \rho gl}{\eta}. \quad (2)$$

For $\Delta \rho \sim 100 \text{ kg m}^{-2}, l \sim 10 \text{ m}, \eta \sim 10^{21} \text{ Pas}$, we obtain $\dot{\gamma} \sim 10^{-12} \text{ s}^{-1}$, which is 7 orders of magnitude smaller than the lower limit of the deformation experiment around $\dot{\gamma} \sim 10^{-10} \text{ s}^{-1}$ (Lockner, 1995). We infer that at small strain rates relevant to the fluid migration in planetary interiors, $\gamma_y$ can become much larger than those obtained from rock deformation experiments, such that it can become comparable to those of agar.

5. Results

5.1. Parameter dependence of the trajectory and the crack shape

We have conducted a series of experiments in the parameter space of the density difference ($\Delta \rho$) and agar concentration ($C$) (Fig. 3(a)), from a total of 85 experiments, we have classified the experiments according to the trajectory type into 5 regimes: I stop, II linear path (2D), III meandering (2D), IV bifurcate (2D), and V pipe-like path (3D), as shown in Fig. 3(a). Here we note that the ”II. linear path (2D)” regime exists in both the large C and small C cases, and is separated by the regimes III and IV. For large C and small $\Delta \rho$ cases, the descent stops at a certain distance away from the injection point, the same phenomenon reported by Taise and Tait (2009). In Fig. 3(b), we classified the experiments according to the descent distance from which we find that there is a general trend of increasing distance of descent, as $\Delta \rho$ increases and $C$ decreases. We next describe each of these regimes in detail.

When the buoyancy is insufficient to fracture the agar, the fluid stops immediately after injection. Fig. 3(a) shows that there is a critical density difference ($\Delta \rho_c$) below which there is no fluid migration, and that $\Delta \rho_c$ increases with $C$.

When $\Delta \rho$ is sufficiently large, buoyancy-driven fracturing is possible and the fluid migrates by forming a crack. Fig. 4 is an example of the shape of a crack for the stiffest agar used in our experiments ($C=0.4 \text{ wt.\%}$), viewed simultaneously from two orthogonal directions. Note that a small fraction of the fluid is left behind forming a planar path which is evident as a faint red region in Fig. 4(a). Such trailing path is a common feature for all of the experiments which we have conducted. The crack is blade-like with a width of $w = 30 \text{ mm}$ (Fig. 4(a)), a thickness of $h \approx 2 \text{ mm}$ (Fig. 4(b)) and a length of $l \approx 16 \text{ mm}$ (see Fig. 6(d) for the definition of these scales). Here $l$ is defined as the height range which appears dark red in Fig. 6(a). Such a blade-like shape is consistent with the previous experiments on buoyancy-driven cracks which used gelatin with a comparable stiffness as we described in Section 3. We define the aspect ratio $a$ of the crack as the ratio of thickness to the width, and for this case it becomes $a = h/w = 0.067$. The hemispheric disk region near the crack tip which is evident in Fig. 4(a) has a volume of $0.7 \text{ cm}^3$. Here when calculating the crack volume from the image, we assumed that the optical distortion by the cylindrical tank can be neglected which we confirmed by inserting a scale in the agar filled tank. Since the volume of the fluid is $1 \text{ cm}^3$, this indicates that about 70% of the fluid is contained in this region. The crack decelerates with time and eventually stops propagating at a distance of $z = 152 \text{ mm}$. Similar blade-like cracks with a trailing linear path form when $0.15 \leq C \leq 0.4 \text{ wt.\%}$ and $\Delta \rho$ is immediately above the threshold for crack propagation.

When $\Delta \rho$ becomes sufficiently large at the intermediate agar concentration of $0.1 \leq C \leq 0.3 \text{ wt.\%}$, a new style of fluid migration emerges. Here the crack does not propagate linearly, but meanders (Fig. 5(a)) or bifurcates (Fig. 5(b)). For these cases, the crack propagates a longer distance than in the agar with a higher concentration. In Fig. 5(a) the total descent distance is $390 \text{ mm}$, and in Fig. 5(b), it is $500 \text{ mm}$. These meandering or bifurcating 2D cracks are observed when $\Delta \rho$ becomes larger than critical.

When the agar becomes even softer ($C \leq 0.08 \text{ wt.\%}$), the fluid no longer meanders or bifurcates, but migrates forming a linear path...
The fluid has a diapiric head followed by a dyke-like trail, and we call this a diapir–dyke hybrid. Although the head is diapir-like, it is actually a bulged crack because it closes at the tail to form a dyke. Fig. 6 shows the examples of such cases for an agar concentration of $0.07 \leq C \leq 0.08$ wt.$\%$. Here views from 3 orthogonal directions are shown for 3 cases with increasing $\Delta \rho$. The descent distances are 440 mm, 500 mm (to the bottom), and 500 mm for Fig. 6(a), (b), and (c), respectively. From the side views, we find that as $\Delta \rho$ increases, the width $w$ and the thickness $\delta$ of the crack head becomes larger, and as a result the crack head becomes increasingly bulged, forming a diapiric head. In detail, in Fig. 6(a) the width of the crack head and the trailing tail is comparable; $w \approx 9$ mm (see a2). On the other hand, the thickness of the crack head $\delta \approx 6$ mm, is larger than that of the tail $\delta \approx 2$ mm (see a1). When $\Delta \rho$ becomes larger, both the width and thickness of the crack head exceed those of the tail. For example in Fig. 6(b), the crack head and tail have widths of $w \approx 16$ mm and $w \approx 14$ mm (see b1), and thicknesses of $\delta \approx 11.8$ mm and $\delta \approx 2$ mm (see b2), respectively. For an even larger $\Delta \rho$, this trend is enhanced. In Fig. 6(c), the crack head and tail have widths of $w \approx 23$ mm and $w \approx 15$ mm (see c1), and thicknesses of $\delta \approx 21$ mm and $\delta \approx 8$ mm, respectively. From the bottom the cracks appear as cusped ellipses, and using the bottom images, we obtain the aspect ratios for the 3 cases as $a = \delta/w = 0.59$ (a3), $a = 0.71$ (b3) and $a = 0.87$ (c3), respectively, which increase with $\Delta \rho$. Such diapir–dyke hybrid with linear paths are observed when $C \leq 0.1$ wt.$\%$.

For the softest agar used in our experiments ($C = 0.06$ wt.$\%$), when $\Delta \rho$ becomes larger than critical, the trailing path becomes pipe-like (3D) and slightly meanders as it descends. Fig. 7 shows such an example at $C = 0.06$ wt.$\%$ and $\Delta \rho = 605$ kg m$^{-3}$. The width of the head becomes $w \approx 29$ mm (see a1) and is even more flattened in the vertical direction. The trailing path has a diameter of $\sim 6$ mm. The head when viewed from the bottom does not have a well-defined cusp. The descent distance is also $500$ mm (i.e., to the bottom) for this case. We note that such pipe-like cracks are becoming similar to creeping plumes rising in viscous fluids (Olson and Singer, 1985), which is also known to leave a small fraction of fluid behind thereby forming fossil conduits.

Here we remark that in the experiments, the injected fluid remains connected to the syringe tip as is evident from Figs. 4 and 5. However because we inject fluids which are miscible with agar, there is no surface tension at the syringe tip which acts to suspend the injected fluid against gravity. Therefore the injected fluid falls freely. We confirmed this by conducting experiments in which we removed the syringe immediately after we injected the fluid. We indeed find that the style in which the fluid descended remained unaffected.

Next we consider the aspect ratio $a$ of the crack head. When calculating $a$, we use 3 images taken from the bottom during the descent, and calculated their average and standard deviation. We use the results for $C \leq 0.1$ wt.$\%$ in which the images from the bottom are large enough for calculating the aspect ratios owing to the longer descent distance. In Fig. 8, we plot the aspect ratios as a function of dimensionless buoyancy $B$ defined as

$$B \equiv \frac{\Delta \rho g l}{G(1 - \nu)}$$

Here $l$ is the length scale of the crack head where we used a constant value of $l = 0.01$ m to evaluate $B$, and $\nu = 0.5$ is the Poisson’s ratio. $l$ varies by $\leq 50\%$ with time and the parameters but its effect is small in a logarithmic plot of Fig. 8. $B$ is primarily controlled by $\Delta \rho$ and $G$. We also note that we take the crack head length, rather than the total length because we consider that the force balance is different in the tail (Roper and Lister, 2007). This is also consistent with the observation that $a$ does not increase as the crack lengthens. Such different force balance in the head and tail is similar to that used to model viscous plumes (Turcotte and Schubert, 2002). The plot shows that the aspect ratios for different $G$ (i.e., agar concentration) and $\Delta \rho$ collapse well, when plotted as a function of $B$. From the power-law fit, we obtain a scaling relation of $a \propto B^{0.14}$. For comparison we consider a
2D buoyant crack in an elastic medium with a length scale of \( l \). If the buoyancy pressure \( \Delta P = \Delta \rho g l \) alone is the origin of crack bulging, then aspect ratio becomes \( a = B \) (Rubin, 1995). A scaling law obtained from our experiments show a much weaker dependence on \( B \) compared to the elastic case, that can be attributed to viscoelasticity. We discuss this in further detail in Section 6.2.

5.2. Velocity of fluid migration

In Fig. 9(a) we plot the distance (\( z \))–time (\( t \)) data for the case in which the fluid migrates in a stiff agar (\( C = 0.08 \) wt.%) with a shape of a blade-like crack. Here two cases with different density differences (\( \Delta \rho = 600, 793 \) kg m\(^{-3} \)) are shown, and we plot the data after the injection of the fluid has been completed. The plots have slopes of <1 indicating deceleration, and their slopes gradually become smaller with time before stopping. We note that a larger \( \Delta \rho \) case has a larger velocity, descends a longer distance and has a smaller deceleration (larger slope). We fit the \( z-t \) data by a power-law function (\( z = c t^n \)) by a least squares method using 5 or more data points immediately after the injection ends, such that the correlation coefficient becomes maximum. The obtained power law exponents are \( n = 0.28 \) and 0.50 for \( \Delta \rho = 600 \) kg m\(^{-3} \) and 793 kg m\(^{-3} \) cases, respectively.

In Fig. 9(b), we plot the data for a soft agar case (\( C = 0.07 \) wt.%), in which the fluid has a shape of a diapir–dyke hybrid. We similarly find
the trend of faster velocity and smaller deceleration with increasing \(\Delta \rho\). We obtain the power law exponents from the \(z-t\) data which are plotted in Fig. 9(c), indicating a similar increase of the exponent with \(\Delta \rho\). In particular, the exponents for \(\Delta \rho = 604 \text{ kg m}^{-3}\) and \(797 \text{ kg m}^{-3}\) cases are \(n = 1.04\) and \(0.93\), respectively, which are larger than those at comparable \(\Delta \rho\) for an agar of \(C = 0.4 \text{ wt.}\%\) described above. We can calculate the velocity at the reference distance of 200 mm, and plot it as a function of \(\Delta \rho\) in Fig. 9(d). Here we show the results for \(0.06 \leq C \leq 0.1 \text{ wt.}\%\). Fig. 9(d) shows that for these values of \(C\), velocity does not depend on \(C\). We fit the data in Fig. 9(d) using a power-law function \(v \propto \Delta \rho^m\). We fit from the smallest \(\Delta \rho\) and select the range at which the correlation coefficient becomes a maximum \((R = 0.98)\) and obtain \(m = 1.56\) using the data for \(\Delta \rho \leq 153 \text{ kg m}^{-3}\). When we fit the data at \(\Delta \rho \geq 190 \text{ kg m}^{-3}\), we obtain a power-law exponent of \(m = 0.56\), indicating a weaker dependence on \(\Delta \rho\).

Using the velocity values, we can evaluate the Reynolds number \(Re = \frac{w \cdot v}{\nu}\) of the flow within the crack, using the characteristic velocity \(v\), thickness \(\delta\) scales, and kinematic viscosity \(\nu\) of the injected fluid. The maximum \(Re\) in our experiments can be estimated by taking the maximum velocity as \(v \sim 0.1 \text{ m s}^{-1}\), \(\nu \sim 6 \times 10^{-7} \text{ m}^2\text{s}^{-1}\) and the thickness of the crack as \(\delta \sim 10^{-2} \text{ m}\), from which we obtain \(Re \sim 2000\). This is smaller than the critical value of \(Re \sim 2000\) for the laminar–turbulent transition in a pipe flow (Tritton, 1988), from which we conclude that the flow in our experiments can be regarded as laminar.
become shorter (small \( l \)) and thicker (large \( \delta \)) as \( \Delta \rho \) becomes larger or \( C \) becomes smaller (see Fig. 10(b)), which is consistent with the curves.

We note however, that in Fig. 3(b), the value of \( l \) which overlaps with most of the cases at low agar concentrations (\( C \leq 0.1 \text{ wt.\%} \)), is \( l \leq 5 \text{ mm} \). This is smaller than the actual length of \( l \sim 10 \text{ mm} \). One possible reason for this difference is the rate-dependence of \( \sigma_y \). At low \( C \), the descent velocity becomes \( v \sim 0.1 \text{ m s}^{-1} \), which corresponds to a shear rate of \( \dot{\gamma} \sim 10 \text{ s}^{-1} \). Since at \( C \leq 0.1 \text{ wt.\%} \), \( \sigma_y \) is rate-strengthening, using a smaller value for \( \sigma_y \) at \( \dot{\gamma} \sim 1 \text{ s}^{-1} \), results in a smaller value of \( l \), which agrees with the experiments.

Here we compare with the result of Taisne and Tait (2009), who conducted a similar experiment using a gelatin with a shear modulus which is comparable to agar at \( C \approx 0.4 \text{ wt.\%} \) and reported that a constant volume crack stops propagating. They interpreted this as a consequence of crack becoming thinner (smaller aspect ratio) as the descent distance becomes longer, which results in a smaller stress available to fracture horizontally. They measured the descent distance \( L_f \) from a series of experiment by changing crack volume \( V \), fracture toughness \( K_c \), and density difference \( \Delta \rho \), to obtain a scaling relation \( L_f \sim \sqrt{V/\rho g} (\Delta \rho/K_c)^{n/6} \). Since \( K_c \) is analogous to \( \sigma_y \) in our experiments, the qualitative dependence of \( L_f \) on \( K_c \) and \( \Delta \rho \) is consistent with our experiments, suggesting a common mechanism.

6.2. Aspect ratio of crack head

First we consider the aspect ratio of a crack in the stiffest agar used in our experiments (\( C \approx 0.4 \text{ wt.\%} \)). For example, in the case shown in Fig. 4, we can calculate the aspect ratio using the front and side images, to obtain \( a = \delta/w = 0.067 \). Since the agar rheology is nearly elastic for this case, we may compare this value with the theoretical aspect ratio \( a \) of a circular (penny-shaped) crack with a radius \( r \), which is expressed as (Heimpel and Olson, 1994)

\[
\frac{a}{r} = \frac{3\sqrt{3}}{2\pi} \left( \frac{\Delta \rho \rho g}{\sigma_y} \right)^{1/2} \tag{5}
\]

Substituting the values \( \Delta \rho = 1793 \text{ kg m}^{-3}, r \approx 15 \text{ mm}, G \approx 1000 \text{ Pa}, \nu = 0.5 \), into Eq. (5), we obtain a theoretical estimate of \( a \approx 0.11 \), which agrees with the measured value within a factor of 2.

Next we proceed to consider the soft agar cases in which the viscoelasticity becomes important. For the case shown in Fig. 6(c) for example, using the bottom image, we obtain \( a \approx 0.87 \). When we substitute \( \Delta \rho = 604 \text{ kg m}^{-3}, r \approx 14 \text{ mm}, G \approx 0.4 \text{ Pa} \) into the elastic model (Eq. (5)), we obtain \( a \approx 0.86 \) which is much larger than the measured value. We consider that such bulged cracks do not form because fracturing at the crack tip occurs before becoming bulged and also because viscous relaxation acts to retain the aspect ratio as \( a \leq 1 \). Such relaxation is also compatible with the relation \( a \approx \dot{\gamma}^{0.14} \) (Fig. 8) for soft agar cases, which has a smaller exponent compared to the elastic case (\( a \approx B \)).

To summarize, the aspect ratios in a stiff agar (\( C \approx 0.4 \text{ wt.\%} \)) agree fairly well with the estimate using the elastic model whereas for soft agars (\( C \approx 0.06 \sim 0.2 \text{ wt.\%} \)), they are much smaller than the estimate. Viscoelasticity need to be taken into account to explain the latter cases.

6.3. Velocity of fluid migration

6.3.1. Distance–time relation

Our experiments indicate that fluids decelerate with time (Fig. 9(a) and (b)). This corresponds to a power-law exponent of \( n \sim 1 \) when we fit the distance–time data by a function \( z = ct^n \).

First, we consider the power-law relation in an elastic medium. An exponent of \( n = 1/3 \) has been predicted by a 2D (\( \sim \)blade-like) numerical model of a constant volume crack by Roper and Lister.
This is a consequence of crack becoming thinner with time, which changes the force balance between the buoyancy and viscous stresses. Similar deceleration has been reported in the experiments of Taisne and Tait (2009), in which they also obtain an exponent close to \( n = 1/3 \) for a blade-like crack. Here we compare with our experiments in a nearly elastic agar at \( C = 0.4 \ wt\% \) in which the fluid with \( \Delta \rho = 600 \ kg \ m^{-3} \) forms a blade-like crack. For this case (Fig. 9(a)), we obtain an exponent of \( n = 0.28 \), which is fairly close to \( n = 1/3 \).

Next we proceed to consider the results for diapir–dyke hybrids at low \( C \) (Fig. 9(b) and (c)), which shows that the exponent exceeds \( n = 1/3 \) as \( \Delta \rho \) increases. Here we note that the crack becomes 3D-like (larger aspect ratio) as \( \Delta \rho \) increases (Fig. 8). It follows that there is a positive correlation between the aspect ratio and \( n \), i.e., as the crack bulges and its aspect ratio becomes larger and 3D-like, \( n \) becomes larger. The same correlation can be confirmed when the medium becomes softer (small \( C \)), under a comparable \( \Delta \rho \). For the case shown in Fig. 4 (\( C = 0.4 \ wt\% \), \( \Delta \rho = 793 \ kg \ m^{-3} \)), the aspect ratio is 0.067 and the power-law exponent is \( n = 0.50 \). On the other hand, for the case shown in Fig. 9 (\( C = 0.07 \ wt\% \), \( \Delta \rho = 797 \ kg \ m^{-3} \)), which has a comparable \( \Delta \rho \) but smaller \( C \) (softer medium), both the aspect ratio (\( a = 0.87 \)) and the power-law exponent (\( n = 0.93 \)) become larger. A positive correlation between the crack shape (aspect ratio) and \( n \), can be interpreted as follows. A diapir–dyke hybrid (large aspect ratio) leaves a smaller fraction of its fluid behind in its trailing tail, compared to a blade-like crack. As a consequence, the stress available for fracturing at the crack tip decreases more slowly compared to a blade-like crack with a small aspect ratio, such that it results in a slower deceleration (larger \( n \)).

### 6.3.2. Velocity–density difference relation

Velocity–density difference relation in soft agars (Fig. 9(d)) showed that the velocities are comparable when the density differences are the same, suggesting that the fracture resistance at the crack tip is irrelevant to the fluid migration. This is in contrast to large agar concentration case (\( C = 0.4 \ wt\% \)), in which for the same density difference (e.g., \( \Delta \rho = 793 \ kg \ m^{-3} \)) the descent velocity (\( v \sim 5 \times 10^{-3} \ m \ s^{-1} \)) is smaller by an order of magnitude. Since the flow can be considered as laminar (Section 5.2), we may apply a channel flow model

\[
\nu = \frac{\dot{\gamma}^2}{12\eta} \Delta \rho \gamma
\]

where \( \dot{\gamma} \) is the channel thickness and \( \eta \) is the dynamic viscosity of the fluid (Rubin, 1995), as the simplest starting model to describe the flow in which the fracture resistance at the crack tip can be neglected. Substituting the experimental values, we find that a narrow channel with \( \dot{\gamma} \sim 0.3 \ mm \) is required to explain the measured velocity of \( v \sim 10^{-3} \ m \ s^{-1} \). However such a narrow channel is smaller than the observed thickness of a few mm. This thickness may correspond to the thickness of the crack tip cavity as shown schematically in Fig. 10(a) and (b). Our experiments also indicate that at small \( \Delta \rho \), velocity scales as \( v \propto \Delta \rho^{0.56} \) whereas the channel flow model predicts \( v \propto \dot{\gamma}^2 \Delta \rho \). One possible explanation for an exponent larger than 1 may be due to \( \dot{\gamma} \) becoming larger with \( \Delta \rho \) as shown in Fig. 10(a) and (b).

At large \( \Delta \rho \), the scaling relation becomes \( v \propto \Delta \rho^{0.56} \), indicating that there is an additional mechanism which acts to inhibit fluid descent into the channel. Here we note that the shear wave velocity estimated using \( G \) for agar concentration of \( C = 0.06 \) to 0.1 \ wt\% is in the range of 0.02 to 0.07 m s\(^{-1}\). Comparing with the velocity in Fig. 9(c), we find that the descent velocity is comparable to or even exceeds the shear wave velocity in the regime of \( v \propto \Delta \rho^{0.56} \). This suggests that shear wave velocity at the crack tip may become a rate-limiting factor for the fluid descent. Further theoretical study is needed to substantiate this interpretation. In addition a series of experiments in which the injected volume and fluid viscosity are varied, are needed to clarify how the velocity can be scaled in terms of dimensionless numbers.

### 6.4. Comparison with previous works around the brittle–ductile transition

First, we compare with the experiments by Hirata (1998), which although have been conducted in a different geometry, used an agar with a similar rheology. In Hirata (1998), air was injected into a Hele–Shaw cell and 3 types of fracturing patterns, namely, viscous fingering, viscoelastic fingering, and single plane cracking, were classified in the parameter space of injection air pressure and agar concentration. Hirata (1998) used an agar with a gel strength (defined by Japan Agricultural Standard) as \( 600 \) to \( 700 \) g cm\(^{-2}\) for a 1.5 \ wt\% agar, a value close to those we used (\( 650 \pm 30 \) g cm\(^{-2}\)). The agar concentration in their experiments was in the range of 0.1 to 0.3 \ wt\%, and overlaps those in our experiments. Our results shown in Fig. 3(a) closely resembles Fig. 6 of Hirata (1998). In detail, the 2D linear trajectory and pipe-like trajectory regime at \( C \leq 0.1 \ wt\% \) corresponds to the viscous fingering, the meandering-bifurcating regime corresponds to viscoelastic fingering, and 2D linear trajectory regime at \( C \geq 0.2 \ wt\% \) corresponds to single plane cracking regime. This similarity indicates that the common mechanics operate both for pressure induced fracturing and buoyancy-driven fracturing.

Next we compare with the studies on blunt fractures. Papanastasiou (1997) calculated the crack shape by including viscous deformation near the crack tip, and showed that this causes the crack to become shorter and wider than purely elastic cracks. This agrees with our results shown schematically in Fig. 10(a) and (b). Hui et al. (2003) studied the condition for crack blunting for a thin crack (length >> width) and proposed that blunting becomes significant when

\[
\sigma_f / E > 2,
\]

where \( \sigma_f \) is the yield stress and \( E \) is the Young’s modulus. This is equivalent to yield strain (Eq. (1)) exceeding the order of 1. In our experiments, crack blunting becomes significant for an agar concentration of \( C \leq 0.1 \ wt\% \). For a rotation rate of 10 rpm (equivalent to \( \gamma = 1 \)–\( 2 \) s\(^{-1}\)), corresponding to the shear rate of the fluid migration experiments, we find that at \( C \leq 0.1 \ wt\% \), \( \sigma_f / E > 0.67 \). This criterion agrees with that of Eq. (7) within a factor of 3.

### 6.5. Implications for buoyancy–driven cracks in planetary interiors

Our experiments suggest that around the brittle–ductile transition, a diapiric head precedes a dyke-like trail, and that it can meander and bifurcate as it ascends. This style of magma transport is one candidate for an agar concentration of \( C \leq 0.1 \ wt\% \), indicating that at \( C \leq 0.1 \ wt\% \), \( \sigma_f / E > 0.67 \). This criterion agrees with that of Eq. (7) within a factor of 3.
attributed to the ductile deformation caused by the diapiric rise of a warm ice and associated melting (e.g., Pappalardo et al., 1998). Interestingly an image from the Galileo (PIA03878) shows that these lenticulae viewed from above have elliptical shapes. Apart from the deformation after they have risen to the surface, our experiments suggest an alternative interpretation in which these are diapir–dyke hybrids.

7. Conclusion

We have found that the buoyancy-driven liquid-filled crack migrates as a diapir–dyke hybrid around the brittle–ductile transition. We have classified the style of fluid migration and descent distance as functions of density difference and agar concentration, and showed that the variation of the style of migration can be explained in terms of the force balance between the buoyancy and yield stress. At the transition there are a number of deviations from the elastic case, such as the relaxation of crack bulging, a slower deceleration rate, and velocity becoming independent on medium rheology.

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Appendix A. Supplementary data

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References