Thermal Interactions Between the Mantle, Outer and Inner Cores, and the Resulting Structural Evolution of the Core

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We review our current understanding of the thermal interactions between the mantle, outer and inner cores, and how they determine the present geomagnetic field pattern and the seismic structure of the core. First, we describe the evolution of the radial structure of the core. The formation of several structures is placed in the context of the Earth’s history. We review the heat flow across the core-mantle boundary, and show a simple model of the inner core growth. We present a model of the initial chemical stable stratification of the core and its subsequent disruption. Model calculations show that viscous compaction efficiently expels liquid from the inner core, and also causes a crust-like structure to form beneath the inner-core boundary because the inner core growth rate gradually decreases. Next, we consider how the radial structure is modified as a result of the lateral variation of heat transfer in the core. The inner core grows in an anisotropic manner as a consequence of the columnar convection in the outer core, which results in a latitudinal variation of heat transfer. This anisotropic growth gives rise to the observed seismic anisotropy in the inner core. In addition to the latitudinal variation, a longitudinal variation of convective heat flux is likely in the outer core because the mantle is thermally heterogeneous. We show, from experimental and theoretical methods, how this modifies the pattern of the outer core flow and the inner core growth. Spatial and temporal characteristics of the geomagnetic field and the longitudinally heterogeneous seismic structure of the inner core can be interpreted in terms of this modification.

1. INTRODUCTION

There are two major solid-liquid interfaces in the Earth’s interior, the core-mantle boundary (CMB) and the inner-core boundary (ICB), at which the mantle, outer and inner cores are coupled. They can be coupled thermodynamically, by transferring heat (and composition) between these layers. Alternatively, they can be coupled mechanically, by the torques which act between them by transferring angular momentum from one layer to another. The two coupling mechanisms mentioned above have very different time scales. Mechanical coupling is involved in relatively short time scale phenom-
ena. It can be instantaneous such as by gravitational locking, to less than $10^4$ years which is the spin-up time of the outer core. On the other hand, thermal coupling has much longer time scales. The thermal core-mantle coupling can be considered as a quasi-steady process for the outer core; the outer core responds to the mantle instantaneously on the time scales of mantle convection, because the dynamical time scales of the outer core are much shorter than those of the mantle. In that sense, thermal (and compositional) coupling is a geological process, and its time scale is determined by the mantle overturn time of the order of $10^8$ y. The thermal coupling between the outer and inner cores is also a geological process, whose time scale is the age of the inner core, of the order of $10^9$ y.

The understanding of thermal coupling is important in interpreting the non-dipole features of the geomagnetic field, which can be the manifestation of the non-axisymmetric flow pattern controlled by the thermally heterogeneous upper and lower boundaries of the outer core (i.e., mantle and inner core). The thermal influence of the outer core on the inner core is important in interpreting the seismic structure of the inner core, because, in our view, the structure is developed through its growth, which is essentially controlled by how the heat is being transferred in the outer core.

In this paper, we review some models of thermal interactions between the mantle, outer and inner cores, and discuss how they are related to observational evidence. In section 2, we describe the radial structure of the core and its evolution in the Earth’s history. We also show how the partially molten structure of the inner core is developed as a result of its growth. In section 3, we describe how this radial structure is modified due to lateral variation of heat transfer. We show that seismic anisotropy of the inner core can be considered as a consequence of anisotropic heat transfer in the outer core controlled by rotation. We also show how lateral thermal heterogeneities on the CMB can control outer core flow and hemispherically varying inner core seismic structure. In section 4, we describe future prospects which are needed to obtain better models for the core.

2. RADIAL CORE STRUCTURE AND ITS EVOLUTION

2.1. History of the Earth’s Core

The structure of the Earth’s core has been evolving throughout the long history of the Earth. Hence the understanding of the history of the Earth’s core is necessary in order to understand various structures in the core.

The Earth formed as a result of the accretion of planetesimals in the primordial solar system. The bombardment of planetesimals released a substantial amount of energy, heating the Earth above the melting temperature of rocks. Molten rock formed the magma ocean, in which metallic components, mostly iron, sank to the center to form the Earth’s core [e.g., Stevenson, 1981, 1990].

As the Earth gradually cools down after the core formation, the inner core solidifies (grows) from the Earth’s center [Jacobs, 1953]. This process has been shown in a number of thermal history calculations [e.g., Gubbins et al., 1979; Stevenson et al., 1983; Buffett et al., 1992, 1996; Sumita et al., 1995; Labrosse et al., 1997, 2001]. As the inner core solidifies, light elements, such as sulfur, oxygen, carbon, or hydrogen [e.g., Jeanloz, 1990; Poirier, 1994] are expelled from the inner core. The release of gravitational energy due to the release of light elements is considered to be essential in driving the Earth’s dynamo [e.g., Gubbins, 1977; Gubbins and Masters, 1979].

In the remainder of this section, we first give estimates of the heat flow across the CMB, which is important in the thermal history of the Earth’s core, and then discuss three consequent one-dimensional models of the structures of the core: the inner core growth, possible initial stable density stratifications in the outer core, and the partially molten structure in the inner core.

2.2. Heat Flow Across the CMB

There are various estimates of the heat flow across the CMB. They are obtained by inferring the cooling rate of the core, by inferring the heat transport by mantle plumes, which may be considered to originate from the CMB, or by inferring the dissipation due to dynamo action.

The CMB heat flow can be estimated if we know the cooling rate of the core as

$$Q_{\text{CMB}} = -C_{\text{eff}} \frac{d\Theta}{dt},$$

where $Q_{\text{CMB}}$ is the heat flow across the CMB, $\Theta$ is the potential temperature of the core, and $C_{\text{eff}}$ is the effective heat capacity, expressed as

$$C_{\text{eff}} = C_{\text{cool}} + C_{\text{latent}} + C_{\text{grav}}.$$
effects from thermal contraction [Stacey, 1992], but the main contribution comes from the three effects above. Since the three heat capacities have similar values of about $2 \times 10^{27}$ J/K, the effective heat capacity $C_{\text{eff}}$ is about $6 \times 10^{27}$ J/K. Note that the values of $C_{\text{latent}}$ and $C_{\text{grav}}$ depend on the difference between the liquidus temperature gradient and the adiabatic gradient $dT_L/dp - dT_{\text{ad}}/dp$, which is highly uncertain. We use the value $2.5 \times 10^{-9}$ K/Pa for the estimate above. Note also that the values of $C_{\text{latent}}$ and $C_{\text{grav}}$ change with time. They are approximately proportional to the inner core radius.

An estimate of the cooling rate is given by assuming that the mantle was completely molten at the formation of the Earth. The mantle solidified very rapidly after the period of heavy bombardment because the heat transport of the molten mantle was very efficient [e.g., Abe, 1997]. The solidification of the mantle dramatically decreased the efficiency of heat transport, and after that the cooling became gradual. If we assume that the temperature at the CMB 4.5 Ga ago was about 4300 K, the extrapolated solidus of pyroilte [Boehler, 2000], and the present CMB temperature is about 4000 K, which is deduced from melting temperature of Fe-O-S system [Boehler, 1996, 2000], the temperature drop in 4.5 Ga is about 300 K. The mean cooling rate thus becomes $7 \times 10^{-8}$ K/y, which gives the heat flow of about $8 \times 10^{12}$ W if we take the value of $4 \times 10^{27}$ J/K as the mean effective heat capacity.

Another clue can be found from the cooling rate of the mantle. The cooling rate of the core becomes similar to that of the mantle if the CMB heat flow strongly depends on the temperature difference of the core and mantle. Estimates of the cooling rate of the mantle are given by calculations of thermal history [e.g., Sleep and Langan, 1981; Christensen, 1985], and have a range of 140 - 350 K / 3.5 Ga. Petrological evidence suggest cooling rates of < 50K/2.7Ga [Campbell and Griffiths, 1992], 120K/3.2Ga [Ohta et al., 1996], ~ 160K/2.7Ga [Abbott et al., 1994], and 300K/3.5Ga [Green, 1981]. These estimates are about the same as those of the cooling rate of the core given above, suggesting that the CMB heat flow is controlled by the temperature difference between the mantle and the core.

A few thermal history calculations which include both the mantle and the core have been carried out [Stevenson et al., 1983; Mollett, 1984; Stacey and Loper, 1984]. Stacey and Loper [1984] used the constraint that the CMB was at the solidus of the mantle 4.5 Ga ago, and that the inner core has grown to its present size. The parameterizations of Stevenson et al. [1983] and Mollett [1984] make the temperature decrease of the core follow that of the mantle. They therefore arrived at the cooling rates similar to the two simple estimates given above, despite the differences in the formulations of the CMB heat flow. The obtained CMB heat flows are in the range of $2 \times 10^{12}$ W [Stacey and Loper, 1984] to $9 \times 10^{12}$ W [Mollett, 1984].

Another type of estimate of the CMB heat flow can be given by assuming that mantle plumes originate from CMB, and that they carry most of the heat flow from the core. Sleep [1990] estimated the heat flow of about $3 \times 10^{12}$ W, whereas Davies [1988] estimated it to be $2.5 \times 10^{12}$ W. This value gives a lower bound of the heat flow because other contributions to the heat flow are neglected. Mantle convection calculations such as by Tackley et al. [1994] give a CMB heat flow of about $7 \times 10^{12}$ W, although the effect of secular cooling is neglected in their calculations.

Yet another estimate is given by Gubbins et al. [1979], who calculated the heat flow necessary to maintain dynamo action to be about $(2 - 5) \times 10^{12}$ W, although this value depends strongly on the type of dynamo model assumed. Similarly, this would give the lower bound on the heat flow, since not all of the heat is dissipated through dynamo action.

To summarize, all the estimates of the CMB heat flow are of the order of several $10^{12}$ W. We use a core heat flow of $(3 - 5) \times 10^{12}$ W as a typical estimate in the discussion below.

2.3. Inner Core Growth

The change of the inner core radius as a function of time is shown in various thermal history calculations. Here we show by a simple argument, that the inner core radius grows approximately proportional to $V$ if the heat flow is constant. The inner core radius grows approximately proportional to $V$ if the heat flow is constant [Gubbins et al., 1979; Buffett et al., 1992, 1996; Sumita et al., 1995; Lister and Buffett, 1998].

When the inner core is very small, we can neglect the effect of latent heat and gravitational energy on the energy budget, which becomes

$$Q_{\text{CMB}} = -C_{OC} \frac{d\Theta}{dt}, \quad (3)$$

where $C_{OC}$ is the heat capacity of the outer core.

Since the temperature of the ICB is at the liquidus, we can relate the cooling rate of the ICB and the change of the inner core radius as

$$\frac{dT_{\text{ICB}}}{dt} = \frac{dT_{\text{ICB}}}{dp} \frac{dp}{dr} \frac{dr}{dt} = -\rho g \frac{dR_{\text{IC}}}{dt} \frac{dT_L}{dp}, \quad (4)$$

where $\rho g$ is the gravitational acceleration at the core.
where \( T_{ICB} \) is the ICB temperature, \( T_L \) is the liquids temperature, \( p \) is the pressure, \( r \) is the radial coordinate, \( R_{IC} \) is the radius of the inner core, \( \rho \) is the density, and \( g \) is the gravitational acceleration. Here we omit the effect of composition on the liquidus, because the composition of the outer core does not change much when the inner core is small. On the other hand, if the temperature gradient of the core is adiabatic, the ICB temperature decrease may be related to the potential temperature change \( d\Theta/dt \) as

\[
\frac{dT_{ICB}}{dt} = \frac{d\Theta}{dt} + \frac{dT_{ad}}{dp} \frac{dp}{dr} \frac{dr}{dt} = \frac{d\Theta}{dt} - \rho g \frac{dR_{IC}}{dt} \frac{dT_{ad}}{dp},
\]

where \( dT_{ad}/dp \) is the adiabatic temperature gradient. Combining these two equations, we get

\[
\frac{d\Theta}{dt} = -\rho g \frac{dR_{IC}}{dt} \left( \frac{dT_L}{dp} - \frac{dT_{ad}}{dp} \right). \tag{5}
\]

Combining equations (3) and (6), and expressing the radial gravity dependence as

\[
g = \gamma r \tag{7}
\]

where \( \gamma = 4\pi G\rho/3 \), we obtain

\[
Q_{CMB} = C_{OC} \gamma \left( \frac{dT_L}{dp} - \frac{dT_{ad}}{dp} \right) \frac{d}{dt} R_{IC}^2. \tag{8}
\]

When the CMB heat flow is constant, this equation be easily integrated to give an explicit expression of the inner core radius as a function of time as

\[
R_{IC} = \left[ \frac{2Q_{CMB}}{C_{OC} \gamma (dT_L/dp - dT_{ad}/dp)} \right]^{1/2}. \tag{9}
\]

This equation gives a rough estimate of the age of the inner core as

\[
\tau_{IC-growth} = (1 \times 10^8 \text{y}) \times \left( \frac{Q_{CMB}}{4 \times 10^{12}\text{W}} \right)^{-1} \times \left( \frac{dT_L/dp - dT_{ad}/dp}{2 \times 10^{-9} \text{K/Pa}} \right). \tag{10}
\]

This estimate indicates that the inner core has grown to its present size in a time scale of the age of the Earth. The corresponding growth rate of the inner core at present is about 0.1 mm/y, which is a rate comparable to or larger than the sedimentation rate of the deep sea sediment.

If we consider a partially molten inner core, the inner core would grow faster, but this effect is small because the melt is efficiently expelled by compaction (section 2.5). The effects which act as additional heat sources (latent heat and gravitational energy) tend to increase the age of the inner core, and more detailed calculations will give an older age. Indeed, with the CMB heat flow of \( 4 \times 10^{12} \text{W} \), Lister and Buffett [1998] and Buffett [2000], for example, obtained the age of \( 4.5 \times 10^9 \text{y} \) and \( 2.2 \times 10^9 \text{y} \), respectively, with the difference between the two being due mainly to the uncertainty in the estimate of \( (dT_L/dp - dT_{ad}/dp) \). Lister and Buffett [1998] also showed that the inner core radius increases as \( t^{1/6} \) when the outer core is stably stratified initially. The problem of the initial stratification is discussed below.

### 2.4. Initial Density Stratification of the Outer Core and its Destruction

In this section we discuss the stable density stratification of the whole outer core which may have existed at the beginning of the Earth’s history [Morrow, 1991; Stevenson, 1992; Kumazawa et al., 1994]. When the Earth formed, the core grew as iron sank in the magma ocean [e.g., Stevenson, 1981, 1990]. The iron should have been in equilibrium with the lowermost mantle, whose condition changes with time. The core grew simultaneously as the size of the Earth increased through the accretion of planetesimals. The pressure and temperature of the lowermost mantle increased with time accordingly. Light elements such as oxygen [e.g., Ohtani and Ringwood, 1984; Ohtani et al., 1984; Kato and Ringwood, 1989] and hydrogen [e.g., Stevenson, 1977; Fukai, 1984, 1992; Yagi and Hishinuma, 1995; Kuramoto and Matsui, 1996; Okuchi, 1997, 1998] are known to become more soluble in iron as the temperature and pressure increase. It follows that the iron accreted later in time may have contained more light materials, causing the core to become stably stratified.

This stratification could have been sufficiently large as to prevent dynamo action. However, the evidence of the magnetic field at 3.5 Ga [McClimhin and Semenayke, 1980; Hale and Dunlop, 1984; Yoshihara and Hamano, 2000] and the magnetic field reversal at 3.2 Ga [Layer et al., 1996] signify that most of the outer core was already vigorously convecting at 3.5 Ga. The initial stable stratification, therefore, should have been disrupted by some mechanism during the first 1 Ga of the Earth’s history.

The mechanisms of the disruption of the stable stratification can be gradual or catastrophic. Gradual mechanisms include encroachment or entrainment [Lister and Buffett, 1998]. Catastrophic mechanisms include tidal resonant of inertial-gravity waves [Kumazawa et
Figure 1. A sketch showing a model where a chemically stratified layer is gradually destroyed by the release of light elements by inner core growth. The interface between the convecting layer and the stratified layer is denoted by $R_s$.

Here we investigate the encroachment mechanism based on the theory of Lister and Buffett [1998]. The situation is shown in Fig. 1. Encroachment is a mechanism in which stratification is gradually disrupted by the convection developing under the stratified layer. The position of the interface between the two layers is determined by the condition that the buoyancies just above and below the interface are the same [Turner, 1973]. Lister and Buffett [1998] examined the movement of the interface between a thermally stratified layer beneath the CMB and a compositionally convecting outer core, but their theory can easily be extended for the case of initial chemical stratification. In the present case, the thickness of the stratified layer decreases as the light element content in the convecting region increases with inner core growth.

Let us assume that the initial radial profile of light element concentration takes the form of

$$C = C_0 + \Delta C \left( \frac{r}{R_{CMB}} \right)^2,$$  \hspace{1cm} (11)

where $C$ is the concentration, $R_{CMB}$ is the CMB radius, and $C_0$ and $\Delta C$ are constants. The functional form is derived as follows. The pressure at the CMB will increase as $R_{CMB}^2$ as the Earth grows. If the solubility is proportional to the pressure, the concentration will vary as $r^2$ accordingly. We can estimate that the temperature effect on solubility would work in a similar way. If the mantle convection is vigorous in the early Earth, the temperature distribution would follow the adiabatic gradient. Then the temperature would be proportional to the pressure. Assuming that the solubility is proportional to temperature, the concentration would also vary as $r^2$.

Let us denote the radius at the bottom of the stable layer as $R_s$ as shown in Fig. 1. If the compositional buoyancy is much larger than the thermal one, $R_s$ is determined by the continuity of the concentration given by

$$C(R_s^+) = C(R_s^-).$$  \hspace{1cm} (12)

The concentration of the stable layer is given by Eq (11), whereas that of the convecting lower layer is given by the conservation of light material as

$$\int_{R_{IC}}^{R_s} C^- 4\pi r^2 \, dr = \int_0^{R_s} \left[ C_0 + \Delta C \left( \frac{r}{R_{CMB}} \right)^2 \right] 4\pi r^2 \, dr,$$  \hspace{1cm} (13)

which becomes,

$$C^- (R_s^3 - R_{IC}^3) = C_0 R_s^3 + \frac{3}{5} \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2 R_s^5.$$  \hspace{1cm} (14)
Here the convecting lower layer is assumed to be well mixed, and has a uniform composition of $C^-$.

Thus the continuity (12) yields

$$C_0 R_s^3 + \frac{3}{5} \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2 R_s^2 \left( \frac{R_s}{R_{CMB}} \right)^2 = C_0 + \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2,$$

which gives $R_s$ as a function of $R_{IC}$. Eq.(15) can be rearranged as

$$\left( \frac{R_s}{R_{CMB}} \right)^5 - \frac{5}{2} \left( \frac{R_{IC}}{R_{CMB}} \right)^3 \left( \frac{R_s}{R_{CMB}} \right)^2 - \frac{5 C_0}{2 \Delta C} \left( \frac{R_{IC}}{R_{CMB}} \right)^3 = 0. \quad (16)$$

When $C_0 \ll \Delta C$, the third term of the left-hand side of Eq. (16) is smaller than the second term unless $R_s/R_{CMB}$ is very small (i.e., beginning of inner core growth), and can be omitted. This leads to

$$R_s = \left( \frac{5}{2} \right)^{\frac{1}{5}} \frac{R_{IC}}{R_{CMB}}, \quad (17)$$

from which we obtain

$$\frac{R_s}{R_{IC}} = \left( \frac{5}{2} \right)^{\frac{1}{5}}. \quad (18)$$

This equation shows that a thick chemically stable layer still remains even now, which is probably not the case. Hence a strongly stratified layer, $C_0 \ll \Delta C$, cannot be disrupted by encroachment.

On the other hand, when $C_0 \gg \Delta C$, the second term of the left-hand side of Eq. (16) is smaller than the third term, and can be omitted. This leads to

$$\frac{R_s}{R_{CMB}} = \left( \frac{5 C_0}{2 \Delta C} \right)^{\frac{1}{5}} \left( \frac{R_{IC}}{R_{CMB}} \right)^{\frac{1}{5}}. \quad (19)$$

We can use the theory of Lister and Buffett [1998] to obtain the initial change of the depth of the stably stratified layer as

$$\frac{R_s}{R_{CMB}} = \sqrt{15 C_0 \Delta C \left( L - \frac{5}{2} C \right)^{-1} \frac{t}{\tau_{\text{therm}}}}, \quad (20)$$

where $L$ and $C$ are non-dimensional constants and $\tau_{\text{therm}}$ is the thermal diffusion time of the core of about $10^{11}$ y (see Appendix for derivation and the definition of the parameters). Here we assumed that the CMB heat flow is equal to the heat flow conducted down the adiabat. The time required for completely destroying the stable layer can be estimated as

$$\tau_{\text{encroach}} = \frac{1}{15} \frac{\tau_{\text{therm}}}{C_0} \left( L - \frac{5}{2} C \right). \quad (21)$$

If we require that this time should be less than 1 Ga, which is implied by the existence of the magnetic field 3.5 Ga ago, we obtain the upper limit for the initial stratification that can be destroyed by encroachment as

$$\frac{\Delta C}{C_0} < 0.2, \quad (22)$$

where we use the parameter values estimated by Lister and Buffett [1998]. This places an upper limit to the compositional gradient in the outer core when the Earth formed, and may serve as a constraint for the early history of the Earth.

### 2.5. Partially Molten Structure of the Inner Core

The Earth’s core contains some light elements such as oxygen, sulfur etc. They depress the melting point of the solid phase and can give rise to the formation of a partial melt between the solidus and liquidus temperatures. Fearn et al. [1981] argued that a thick partially molten region exists in the inner core because of compositional super-cooling, i.e., inhibition of solidification from excess light element content. If the ICB were flat, the liquid-side of the growing solid-liquid interface would become supercooled because light elements diffuse slowly and depresses melting point. As a result, the growing interface becomes dendritic, and a partially molten layer develops. Fearn et al. [1981] speculated that the partially molten region would extend to the center of the inner core. However, Loper [1983] estimated that the partially molten region would become thinner when convection of the liquid phase is considered.

Sumita et al. [1996] studied the viscous compaction of the solid matrix of a partial melt in a growing inner core and showed that the compaction efficiently expels melt from the inner core. They neglected thermal and compositional effects, and investigated a one-dimensional sedimentary compaction problem in detail. In the Earth’s core, the sedimentation velocity (i.e., solidification rate) $V_0$ is much less than the Darcy velocity defined as

$$V_D = \frac{K}{\eta_f} \Delta \rho g, \quad (23)$$

where $K$ is the permeability, $\eta_f$ is the viscosity of liquid iron, and $\Delta \rho$ is the density difference between the solid
and liquid iron (i.e. inner and outer cores). In that case, the mushy layer, where melt fraction is large, becomes very thin. Its thickness, given by

$$\delta_{\text{mush}} = \sqrt{\frac{\eta_s V_0}{\Delta pg}},$$  \hspace{1cm} (24)$$

where $\eta_s$ is the viscosity of the solid iron, becomes very small, of the order of 10 m when $\eta_s$ is $10^{16}$ Pa s. It shows that the compaction is very effective.

Sumita et al. [1995] included thermal and compositional effects in their calculation, but the essential features were the same as the purely mechanical case. Fig.2 shows a result of their numerical calculation illustrating how the melt fraction and temperature within the inner core decreases as the inner core grows [Sumita et al., 1995]. Here, coupled equations of mechanical compaction, energy and composition conservation were solved, and the melt fraction was determined from the binary eutectic phase diagram assumed for the inner core. We assumed no internal heat sources, no gravitational energy release, an adiabatic temperature profile in the outer core, and a CMB heat flow of $3 \times 10^{12}$W. The radial profile of melt fraction is characterized by three layers. At the top is the mushy layer, which is a thin layer beneath the ICB where the melt fraction decreases sharply. Then there is a transition layer of low melt fraction, followed by a thick layer of constant melt fraction which extends to the center. In the mushy layer, from the combined effect of the deformation of solid matrix and slow inner core growth, the melt is efficiently expelled from the inner core. In the region of constant melt fraction, deformation is limited, and the fluid is expelled by buoyancy driven permeable flow. The melt fraction is depth independent in this region because the gravity is proportional to radius [Sumita et al., 1996]. The region of small melt fraction between these two regions forms because the melt is more efficiently expelled from the inner core for slower growth rates in the recent times. This crust-like region, or a seismic low velocity zone beneath it, may correspond to the seismically detected discontinuity at about 100 km below the ICB [Souriau and Souriau, 1989].

The possible existence of the “crust” and a larger melt fraction zone beneath it resembles the structure near the Earth’s surface and suggests that plate tectonics might be operating in the inner core. However once the crust subducts, the “plate tectonics” would cease because the crust would not be reproduced. This crust results from gradual decrease in the inner core growth rate, and hence rapid reproduction cannot be expected. It would be interesting to investigate seismologically whether the crust-like region exists or is already disrupted.

2.6. Thermal and Compositional Structure of the Inner Core

Thermal structure of the inner core (Fig.2b), shows that the temperature gradient within the inner core is smaller than the adiabatic temperature gradient. This implies that thermal convection cannot occur in the inner core. Temperature gradients calculated by considering heat conduction alone have also shown that it is less than the adiabat [Yoshida et al., 1996; Yukutake, 1998; Buffett, 2000]. This result is a consequence of the
slow inner core growth rate which allows a long time to cool, and of the large thermal conductivity of metallic iron. Advection of heat by permeable flow has an additional effect in reducing the temperature gradient. However this is estimated to be less important as compared to conduction, and can be evaluated from the Péclet number,

$$P_e = \frac{V L}{\kappa}.$$  

(25)

If we take $L \sim 10^6\text{m}$ and $V \sim 10^{-12}\text{m/s}$, a typical velocity of permeable flow, we find $P_e \sim 10^{-1}$.

On the other hand, advection of light elements by permeable flow is generally much more efficient than compositional diffusion through the liquid, because of small diffusion coefficient of light elements such as oxygen and sulfur, in liquid iron of $D \sim 10^{-8}\text{m}^2/\text{s}$ [Iida and Guthrie, 1988], resulting in a compositional Péclet number, $P_{ec} = VL/D \sim 10^2$. Hydrogen is an exception, however, and has a large diffusion coefficient $D \sim 10^{-4}\text{m}^2/\text{s}$ [Iida and Guthrie, 1988]. For this large diffusion coefficient, compositional super-cooling, would not occur according to the condition of Fearn et al. [1981]. In addition, the tendency for compositional cooling is diminished by the partition coefficient of hydrogen close to unity [Fukai, 1992; Okuchi, 1997]; i.e., considerable amount of hydrogen can enter in the inner core, and not much hydrogen is released upon freezing of the inner core. Hence, if the major light element in the core is hydrogen, the inner core would be completely solid with a sharp boundary.

3. LATERAL VARIATION OF CORE STRUCTURES

In the previous section, we have considered the inner core growth under spherical symmetry. We now proceed to consider how this is modified by the convective pattern of the outer core which is strongly controlled by rotation and mantle structure as we show below.

3.1. Anisotropic Growth of the Inner Core

Convection in the outer core is strongly controlled by rotation and its spherical geometry. Busse [1971] proposed from a linear theory that convective pattern in a rapidly rotating spherical shells would form columns aligned parallel to the rotational axis as a result of geostrophic balance (i.e., Taylor-Proudman theorem). Several non-linear numerical calculations [e.g., Zhang, 1991, 1992] and laboratory experiments [e.g., Busse and Carrigan, 1970] have confirmed the two dimensional nature of convective pattern. Because columnar cells are tangential to the inner core at the equator, the heat flow is expected to become anisotropic, becoming largest at the equatorial regions. This was first shown for weakly non-linear calculations [Zhang, 1991, 1992], and was found to be valid even under the presence of a magnetic field [Olson et al., 1999] as well as turbulence [Sumita and Olson, 2000].

Yoshida et al. [1996] proposed the consequence of the anisotropic heat flow on the inner core growth, as shown schematically in Fig.3. The inner core would grow predominantly at the equatorial region, but because of the density difference between the outer and inner cores, the inner core would isostatically deform back to its spherical shape. The deformation time scale can be estimated as

$$\tau_{deform} = \frac{\eta_s}{\Delta \rho g R_{IC}} = 3 \times 10^4 y \left( \frac{\eta_s}{10^{21}\text{Pa} \cdot \text{s}} \right).$$  

(26)

The short time scale even for an inner core viscosity as large as $10^{21}\text{Pa} \cdot \text{s}$ indicates that isostatic adjustment is instantaneous as compared to the time scale for inner core growth, so the inner core remains close to its hydrostatic shape. Since the anisotropic growth continues throughout the Earth's history, the inner core would always deviate a little away from the hydrostatic state. As a result, deviatoric stress would form, which has a sense of pull in the direction of the axis of rotation, and push in the equatorial regions. This deviatoric stress would
Figure 4. Strata in the inner core, joining the points which have solidified at the same time [Yoshida et al., J. Geophys. Res., 101, 28085-29103, 1996, copyright 1996 by the American Geophysical Union]. The inner core is assumed to grow proportional to $\frac{x}{r}$ and predominantly at low latitudes as shown in Fig.3. Each line corresponds to the strata at units of 1/10 of the inner core age. Inner core strata form approximately parallel to the rotational axis, and the iron which has solidified at an early stage accumulates near the rotational axis.

affect how the crystals align as we shall discuss in section 3.2. One consequence of the anisotropic growth is shown in Fig.4. Here, each lines indicate isochrons joining the points which have solidified at the same time. If the inner core grows isotropically, then the isochrons should form concentric spheres. However, in the case of dominant growth at the equator, the isochrons become nearly parallel to the rotational axis, and the iron which has solidified at the early stage accumulates near the rotational axis.

3.2. Seismic Anisotropy of the Inner Core

Large elastic anisotropy has been observed seismologically in the inner core [for reviews see Song, 1997; Creager, 2000; Tromp, 2001] and various causes have been proposed [for reviews see Yoshida et al., 1998; Buffett, 2000]. The anisotropy is transversely isotropic with its symmetry axis being the rotation axis. P-waves traversing through the inner core is fast in the north-south direction, and slow in the equatorial direction.

This type of anisotropy appears as a natural consequence of the anisotropic growth explained above. Yoshida et al. [1996] proposed that preferred orientation of iron crystals occurs due to the flow induced by the anisotropic growth. This was calculated using a preferred orientation theory by Kamb [1959] and the elastic constants of Stixrude and Cohen [1995]. According to Kamb [1959]'s theory, preferred orientation forms by recrystallization under stress, so that it minimizes the elastic strain energy. Yoshida et al. [1996] showed that for a deformation caused by anisotropic growth, deformation should be primarily governed by diffusion creep with Newtonian rheology because the stress level is very low, and argued that Kamb [1959]'s theory would be applicable. The elastic constants of Stixrude and Cohen [1995] was calculated at a core pressure but at 0 K. Recently Steinle-Neumann et al. [2001] obtained the elastic constants of hcp iron at core pressures to temperatures as high as 6000 K. They found that the axial ratio $c/a$ increases with temperature, and as a result, the compressional wave becomes faster in the $a$-axis compared to the $c$-axis, in contrast to the results of Stixrude and Cohen [1995]. Using their elastic constants, we calculate preferred orientation of iron crystals in the inner core following the method described in Yoshida et al. [1996]. Fig.5 shows the calculated preferred orientation with the effect of compaction dominating at the uppermost 60 km. As can be seen from this figure, the $a$-axis, the fast P-wave direction of hcp iron, aligns parallel to the rotation axis in most of the region of the inner core. On the other hand, in the region dominated by compaction, the $a$-axis aligns in the radial direction because the principal stress axis is in the direction of gravity. As a result, P-waves traversing through the inner core would be fast in the north-south direction in most part of the inner core, and the axial anisotropy would be absent in the shallow parts, which is consistent with some seismic observations [e.g., Song and Helmberger, 1995, 1998; Creager, 2000; Ouzounis and Creager, 2001]. It is also important to note that a perfect alignment would not be required to explain the inner core seismic anisotropy of $\sim 3\%$, because the degree of P-wave anisotropy of single crystal hcp iron obtained by Steinle-Neumann et al. [2001] is 13%. We also infer from the isochrons shown in Fig.4 that the anisotropy would be largest at the region close to the rotation axis, because the iron crystals in this region would have been under stress for the longest time, consistent with seismic observations by Romanovicz et al. [1996].

There may be conditions where the heat transfer would become larger at the poles, such as the case when a strong toroidal magnetic field inhibits convection out-
Figure 5. An example of the preferred orientation of iron crystals in the inner core calculated using the method of Yoshida et al. [1996] and using the elastic constants of Steinle-Neumann et al. [2001]. The mechanism of preferred orientation is assumed to be pressure solution. Grain boundary migration gives a similar result. The longer side of the rectangles represents the c-axis of an hcp crystal. We assumed that compaction dominates in the top 150 km beneath the ICB.

side the inner core tangent cylinder. We remark that even under such situations, we obtain the same pattern of preferred orientation, because the principal stress axis is the same. However there may be a stronger anisotropy near the equatorial plane because the crystals would have been under stress for a longer time.

There are other possible mechanisms for the formation of preferred orientation. In usual experimental situations, plastic strain energy is much larger than elastic strain energy, and Kamb’s theory is not applicable. The very low stress level and the long time scale envisaged by Yoshida et al. [1996] are quite different from experimental conditions and Kamb’s theory may still be applicable, but it is of interest to consider other mechanisms as well.

Jeanloz and Wenk [1988] first considered dislocation glide as the dominant deformation mechanism in the inner core. However they assumed thermal convection driven by internal heat generation, and did not provide a reason why the flow should be axisymmetric. The preferred orientation under dislocation glide is determined by the primary slip plane and the flow pattern in the inner core. Poirier and Price [1999] proposed a method of determining the primary slip plane from elastic constants and stacking fault energies, and demonstrated that basal plane should be the primary slip plane for zero temperature ε-iron. The finite temperature effect calculated by Steinle-Neumann et al. [2001] reinforces the preference for basal slip with larger c/a ratio and smaller c66/c44. Wenk et al. [2000a, b] also clarified the preferred orientation of hcp iron under this mechanism. One of their important findings is that basal slip is dominant in determining the overall orientation even if prismatic slip is favored over basal slip. As a consequence, when a simple shear predominates, the a-axis becomes parallel to the shear plane. High pressure deformation experiments show that when the uniaxial strain predominates, the c-axis aligns in the axis of compression, and this alignment can be interpreted to be due to basal slip [Wenk et al., 2000b]. Wenk et al. [2000a] assumed a poloidal degree 2 thermal convection in the inner core with a downwelling at the poles and upwellings at the equator, though we doubt its existence because the temperature in the inner core is likely to be subadiabatic as discussed above. They showed that the c-axis aligns in the direction of the rotation axis because of the strong shear near the equatorial plane. On the other hand, for the flow driven by aspherical growth [Yoshida et al., 1996], the shear in the equatorial region is not strong and uniaxial strain is dominant. Therefore if we apply this alignment mechanism to the case of a predominantly equatorial growth, we would obtain the alignment of the a-axis parallel to the rotation axis, and vice versa for the polar growth. Assuming the elastic constants of Steinle-Neumann et al. [2001], the equatorial growth is in accordance with the seismic observations.

Maxwell stress due to the magnetic field may be important in driving convection in the inner core. Karato [1999] recently argued that the Maxwell stress would be the most important driving force. However, his model does not work as it is because he did not consider the pressure and buoyancy forces, which would balance the Lorentz force. Buffett and Bloxham [2000] examined the balance among the magnetic, pressure and buoyancy forces, and showed that complete equilibrium cannot be attained because of the incompatibility between thermodynamic and hydrostatic equilibrium conditions at the ICB. Thus a viscous flow would occur, but they showed that it is weak and confined to the region near the ICB. Buffett and Wenk [2001] next used the elastic constants by Steinle-Neumann et al. [2001], and considered the azimuthal Lorentz force to be important, and calculated the resulting elastic anisotropy. Because of the azimuthal shear, the a-axis becomes parallel to
the rotation axis, and this is consistent with seismic observations if we adopt the elastic constants of Steinle-Neumann et al. [2001]. Buffett and Bloxham [2000]'s result also implies that heat transfer in the outer core is important because of thermodynamic equilibrium condition at the ICB. Hence, if we are to modify Karato [1999]'s model properly, appropriate heat transfer in the outer core is needed to maintain the inner core away from its hydrostatic shape from solidification and melting. Flow pattern in the outer core determines both the heat transfer and the Maxwell stress pattern at ICB, and their relationship is not yet clarified.

3.3. Effect of Thermally Heterogeneous Mantle

Apart from the rotational control of heat transfer, it is also likely that the core flow is controlled by a thermally heterogeneous mantle, which was first suggested by Jones [1977]. Gubbins and Richards [1986] noted the correlations between the structure of the lowermost mantle, and persistent features of the CMB. If some features persist for a period longer than the time scales of the fluid motion in the outer core, it implies that they should be produced under the influence of the mantle, because otherwise the geomagnetic features would drift eastward or westward. Bloxham and Gubbins [1987] suggested that the stationary flux lobes beneath North America and East Asia is the result of thermal core-mantle interaction. Paleomagnetic data up to 5 My ago have also indicated that these features are persistent (for a review, see Gubbins, [1997]; but see also Kono et al. [2000], for the problems of previous analyses.) It has also been pointed out that non axial dipole components of the geomagnetic field pattern in the Pacific and Atlantic hemispheres have different characteristics [Johnson and Constable, 1998; Walker and Backus, 1996]. Thus geomagnetic data up to 5 My ago do suggest that the geomagnetic field pattern is being controlled by the mantle.

We first note that the convective heat flux can be a small fraction of the total heat flux (i.e., the sum of convective and conductive heat flux) across the CMB. This is because the heat flux which is conducted down the adiabat is large. Assuming constant material properties, the adiabatic temperature in the core is expressed as

$$T = T_{ICB} \exp\left\{-\beta(r^2 - R_{ICB}^2)\right\}$$  \hspace{1cm} (27)

with $\beta = \alpha r^2 / 2C_p$, where $\alpha$ is the thermal expansion coefficient, $C_p$ is the specific heat and $\gamma$ is defined in Eq.(7). Taking $T_{ICB} = 4900K$, the melting point of iron [Boehler, 2000], we find that $T_{CMB} \approx 4000K$ and the heat flux at the CMB which is conducted down the adiabat as $q_{ad}^{CMB} = 18\text{mW/m}^2$. A CMB heat flow of $Q_{CMB} = 3 \times 10^{12}\text{W}$ is equivalent to a heat flux of $q_{CMB} = 20\text{W/m}^2$, and this implies that the convective heat flux in the core can be as small as 1/10 of the total heat flux, or even negative (subadiabatic) as pointed out by Loper [1978].

The smallness of convective heat flux implies its large lateral variation, which may amount to the order of the mean heat flux ($\delta q_{CMB} \sim q_{CMB}$), because we can expect a lateral temperature variation of the order of 1000 K in the mantle, whereas the temperature drop across the D" layer is also of the order of 1000 K [Boehler, 2000]. This means that the lateral variation of convective heat transfer becomes quite large. For example, if the convective transfer is about a tenth of the total heat flux ($q_{conv}^{CMB} \sim 0.1q_{CMB}$), this would lead to a lateral variation of the convective heat transfer of about a factor of 10:

$$\delta q_{conv}^{CMB} \sim 10q_{conv}^{CMB}. \hspace{1cm} (28)$$

The situation is a little different at the ICB. The heat conducted down the adiabat is not a major part of the total heat flux as we show below. The adiabatic temperature gradient, derived from Eq. (27), is an increasing function of radius, and as a result the adiabatic temperature gradient is smaller at the ICB, giving $q_{ad}^{ICB} \sim 0.4q_{ad}^{CMB}$. On the other hand, the total heat flow at the ICB may be approximated as

$$Q_{ICB} = -C_{latent} \frac{d\Theta}{dt}. \hspace{1cm} (29)$$

Hence the heat flux at the ICB is given by

$$\frac{q_{ICB}}{q_{CMB}} = \frac{Q_{ICB}}{Q_{CMB}} \frac{4\pi R_{CMB}^2}{4\pi R_{ICB}^2} = \frac{C_{latent} R_{CMB}^2}{C_{eff} R_{ICB}^2} \sim 3. \hspace{1cm} (30)$$

As a result, we have

$$\frac{q_{ICB}}{q_{CMB}} \sim 0.1 \frac{q_{ad}^{CMB}}{q_{ICB}}, \hspace{1cm} (31)$$

and in contrast to CMB, the convective heat transfer is a major part of the ICB heat flux. Consequently, if the lateral variation of convection extends deep into the core, we would expect a large lateral variation of heat flux at the ICB, which can result in correspondingly large variation of inner core growth.

Because the core has a flow velocity which is faster than that of the mantle by about 5 orders of magnitude, the core would respond instantaneously to the thermal anomaly of the mantle. If we regard the D" layer as a thermal boundary layer for simplicity, we would expect a high heat flow beneath the cold mantle and vice versa. The resulting flow pattern in the core is, however, com-
Figure 6. A photographic image of the planform of the convective pattern with a heterogeneous CMB heat flux [Reprinted with permission from Sumita and Olson, 1999. Copyright 1999 American Association for the Advancement of Science]. $Ra/Ra_c = 24$ and $(\text{peak CMB heat flux})/\langle\text{mean CMB heat flux}\rangle = 95$. Ekman number $Ek = 4.7 \times 10^{-6}$. The location of the heater is indicated by a white rectangle. Rotation is counterclockwise. The radially spiraling structure is the front, along which a geostrophic jet flows from the CMB to the ICB.

Complicated by sphericity, rotation and the magnetic field, in the following sections, we first describe an experimental model of thermal core-mantle interaction, and then compare it with theoretical (analytical and numerical) models.

3.4. Experimental Model of Thermal Core-Mantle Interaction

There have been a few rotating convection experiments with heterogeneous temperature or heat flow boundaries [Hart et al., 1986a,b; Bolton and Sayler, 1991; Sumita and Olson, 1999, 2002]. In particular, experiments using rapidly rotating spherical shells have the correct geometrical configuration and are capable of achieving low Ekman number and high Rayleigh number conditions that are similar to the condition in the outer core [Sumita and Olson, 2000]. Thermal convection in such experiments is characterized by fine-scale two-dimensional plumes (geostrophic turbulence) which are advected westward by the zonal flow driven by the Reynolds stress.

Sumita and Olson [1999] modeled large lateral variations of convective heat flux in laboratory experiments at $Ra/Ra_c < 50$, with a heterogeneous CMB heat flux that has a peak value of up to 100 times its mean. Fig.6 shows an example of how the heterogeneous heat flux affects the convection. Under a homogeneous heat flux boundary condition, the zonal flow is always westward [Sumita and Olson, 2000]. However with a heterogeneous CMB heat flux, an eastward flow was generated from the high heat flux region. When the heat flow at the anomaly (i.e., heat flux $\times$ size of the anomalous region) exceeded the heat flow of the surrounding region (i.e., heat flux $\times$ size of the surrounding region) [Sumita and Olson, 2002], we find that these zonal flows flowing in the opposite directions converge, and form a sharp stationary front, along which a geostrophic (2D) jet flows from CMB to ICB (Fig.6). The large-scale flows can be understood by the balance between vortex stretching and buoyancy, with a large eastward phase shift relative to the large heat flux region resulting from advection (see next section for details). Apart from the large-scale flow, there are also fine-scaled radial flows such as plumes and jets (fronts), where non-linear effects become important. These radial flows advect the anomalous heat towards the inner core, thus allowing the thermal interaction to extend across the shell.

Based upon the experiments, Sumita and Olson [1999] deduced a core flow model as shown in Fig.7. Here, a high heat flow region is assumed to exist beneath east Asia, corresponding to a significant seismic high velocity region near the CMB common to many seismic tomography models [for a recent review see Garnero 2000]. The model of Sumita and Olson [1999] shows that there are basically two regions beneath the CMB: a low pressure region with a cold eastward flow originating from east Asia driven by the high heat flux region, and a high pressure surrounding region with a westward mean flow. This zonal flow pattern is similar to that obtained from geomagnetic secular variations under the tangentially geostrophic approximation [e.g., Bloxham, 1992], which is consistent with the 2D flows of the experiment. According to this interpretation, the geomagnetic westward drift is the result of the warm mean westward flow, which is inhibited by the cold eastward flow in the Pacific that originates from the high heat flow region in east Asia. The hemispherical dichotomy of the outer core temperature has been previously inferred from geomagnetic secular variation [Bloxham and Jackson, 1990] and seismology [Tanaka and Hamaguchi, 1993], and is consistent with this model.

The experiments also showed that a high convective heat flux region exists at the region immediately west of the front, which would lead to a hemispherical di-


Figure 7. A schematic diagram of the model of core-mantle thermal interaction based on laboratory experiments [Reprinted with permission from Sumita and Olson, 1999. Copyright 1999 American Association for the Advancement of Science]. This is an equatorial planform of the Earth's core as seen from the north pole showing a CMB patch with a high heat flux (hatched area). Rotation is counterclockwise. The numbers indicate longitude. Black (white) headed arrows indicate warm (cold) radial flows. H and L indicate the high and low pressure regions, respectively. The shaded region at the ICB is the inferred region of fast solidification.

The experimental model shows a dichotomy of outer core flow beneath CMB and a dichotomy of inner core growth rate. Because of the eastward spiraling nature of the front, the pattern of inner core dichotomy is shifted east relative to the flow dichotomy beneath the CMB (In Fig.7 this is about 60°). The angle of phase shift is an increasing function of the heat flow at the high heat flux region because of larger advection of heat. This implies that it is possible to constrain the relative magnitude of anomalous total heat flow, provided that the size of the high CMB heat flow region is well constrained by seismic tomography.

If this model is applicable to the Earth, it can have other implications. For the lateral variation of inner core structure to form, the pattern of outer core flow and the inner core should remain fixed relative to the mantle for the time scale of inner core growth, if Yoshida et al. [1996]’s model is valid. This rules out the long-term inner core rotation relative to the mantle. The gravitational torque between the mantle and the inner core [Buffett, 1996] would be the mechanism to prevent the relative rotation. It also indicates that there must be a mechanism for the pattern of lateral heterogeneity at the CMB to remain persistent for about ~1Ga, the time scale required for inner core growth to produce hemispherical dichotomy.

3.5. Theoretical Model of Thermal Core-Mantle Interaction

There have been a number of numerical studies on thermal core-mantle coupling and how it affects the geomagnetic field pattern [Zhang and Gubbins, 1992, 1993, 1996; Yoshida and Hamano, 1993; Sun et al., 1994; Olson and Glazmaier, 1996; Sarson et al., 1997; Glazmaier and Roberts, 1997; Glazmaier et al., 1999; Bloxham, 2000a, b; Yoshida and Shudo, 2000; Gibbons and Gubbins, 2000] but a complete understanding including the non-linear and magnetic effects have not been achieved yet. However, the fluid mechanics of linear and weakly non-linear cases are relatively well understood.

Yoshida and Shudo [2000] carried out a detailed study of a linear response of the outer core flow to a thermally heterogeneous mantle. The basic equations they used are for no magnetic field and no basic flow; the velocity is determined by the thermal wind equation

\[ 2\Omega \frac{\partial \mathbf{v}}{\partial z} = -ag \times \nabla T, \]

and the temperature is governed by the Laplace equation

\[ \nabla^2 T = 0, \]

where \( T \) is the temperature, \( \Omega \) is the rotation vector, \( \mathbf{v} \) is the velocity and \( z \) is the coordinate along the rotation axis. The thermal wind equation is obtained by taking the rotation of the equation of motion, and represents the balance of vorticity generation. They found rigorous analytical solutions for the inviscid problem [Yoshida, in preparation]. An example of such solutions is shown in Fig.8. It is for simulating the situation in Sumita and Olson [1999]’s experiment (Fig.6). Here a cold region with a Gaussian profile is imposed at the CMB at a longitude centered at 0°, with a half width of 30°. The obtained flow has a large-scale pattern similar to the experiment, with an eastward phase shift relative to the thermal anomaly. It consists of a cyclonic circulation near the cold region and the surrounding anticyclonic circulation. The equatorial section of the flow pattern...
is easily understood by the quasi-geostrophic model of Yoshida and Hamano [1993], because the geostrophic component dominates in the equatorial section. In the region where the temperature increases eastward, anticyclonic vorticity is generated due to buoyancy. In order to balance the vorticity generation, the geostrophic flow should move inward to produce cyclonic vorticity. That is why downwelling occurs to the east of the cold region.

The similarity of the large-scale flow between the linear theory and the experiment may seem surprising, if one considers the large Péclet number $Pe = VL/\kappa \sim 1800$ of the experiment, where the shell thickness is taken to be $L$ [Sumita and Olson, 2000]. This may be because of the thermal convection occurring in the experiment. Its turbulence would give rise to a large effective eddy diffusion, thereby reducing the effective Péclet number.

Nonlinear effects are of course prominent in the experiment. First, jets and fronts are formed. Second, the eastward phase shift is large and shows spiraling. Also the phase shift increases with the magnitude of the thermal anomaly. These phenomena result from the advective term. Advection is important for fine-scaled features such as plumes whose velocity and length scales are properly understood when non-linear terms are considered [Cardin and Olson, 1994; Aubert et al., 2001; Sumita and Olson, 2002]. Improvement in theoretical understanding is necessary in order to understand these nonlinear features well.

The Lorentz force has been omitted in the discussion so far, and the flow was assumed to be nearly geostrophic. Although it seems that the flow in the core is characterized by a columnar pattern, as is supported by the presence of equatorially symmetric flux lobes [Gubbins and Bloxham, 1987], Lorentz force is generally considered to be important for the force balance (Elsasser number $\sim O(1)$). In a linear response theory, Yoshida and Hamano [1993] showed that the eastward phase shift is suppressed when the Elsasser number $> O(1)$. The apparent consistency of interpreting the core structure by an eastward phase shift may indicate that the advective effects are large enough as to produce a net eastward phase shift.

4. FUTURE PROSPECTS

We describe several future prospects that are particularly important to evaluate the different models and for quantitative comparison with the observation.

A better understanding of the properties of iron at core conditions is a key to an improved quantitative comparison between the models and seismic observations. Evaluation of properties such as diffusion coefficient in the fluid and solid iron would be helpful to understand deformation mechanism in the inner core.

A detailed study on the pattern and variation of heat transfer in the outer core is important to evaluate several candidates of inner core anisotropy. The question of whether heat transfer has a maximum at the equator or at the poles is important to several anisotropy models. If the elastic constants of Steinele-Neumann et al. [2001] is adopted, solidification texturing of hcp-alloy [Bergman, 1997] requires a polar dominant heat transfer. As already discussed, for a dislocation glide mechanism [Wenk et al., 2000a, 2000b] with a flow driven by anisotropic growth [Yoshida et al. 1996], an equatorial dominant heat transfer is compatible. The amplitude of the variation is important for the kinetics of preferred orientation. For example, it affects the strain energy required to drive reorientation by recrystallization under stress [Yoshida et al., 1996] by its square. The weakly non-linear calculations of Zhang [1991] indicate that the amplitude of heat flux variation with latitude increases with Rayleigh number. Strong field dynamo calculation by Olson et al. [1999] indicates a factor of 7 larger heat transfer at the equator as compared to the poles. There is, however, no systematic parameter study on this problem, which will be needed in the future.
3D seismic tomographies near the CMB with a better spatial resolution in the lateral and radial directions are important to constrain the size of the heterogeneity and the radial temperature gradient distribution, respectively. The amplitude of the heterogeneity is needed to constrain the lateral contrast of heat flux. Since CMB is also known to be a place of chemical heterogeneity, using both P and S waves [e.g., Wysession et al., 1999] would also become important to separate the thermal and compositional effects on seismic anomaly.

In this paper, we showed that thermal interactions between the mantle, outer and inner cores can explain a number of major dynamical structures in the core. The inner core growth is sensitive to how heat is being transferred by convection in the outer core. This implies that the inner core structure can be a good recorder of the heat flow pattern and the thermal history of the outer core and mantle. A continued cross-disciplinary research of the thermally coupled mantle, outer and inner cores should give us a better understanding of how the Earth’s interior operates as a system.

APPENDIX: DERIVATION OF EQ.(20)

Here we briefly describe the derivation of Eq.(20). The derivation is based on the theory of Lister and Buffett [1998], but modified for the chemical stratification.

Under the hydrostatic approximation, pressure in the core can be expressed as

\[ p(r) = p_0 - \Delta p \left( \frac{r}{R_{CMB}} \right)^2, \]  

(A1)

where \( p_0 \) is the pressure at the center of the Earth, and \( \Delta p \) is defined as

\[ \Delta p = \frac{1}{2} \gamma \rho R_{CMB}^2. \]  

(A2)

The radial temperature profile in the convecting layer \( T^- \) is then given by the adiabatic relationship as

\[ T^-(r, t) = \Theta^-(t) - \frac{dT_{ad}}{dp} \Delta p \left( \frac{r}{R_{CMB}} \right)^2, \]  

(A3)

where the adiabatic temperature gradient \( dT_{ad}/dp \) is assumed to be constant, and \( \Theta^- \) is the potential temperature of the convecting lower layer. The potential temperature is defined here as the temperature which the fluid element would have if it were brought adiabatically to the center of the Earth. The temperature distribution of the stratified upper layer \( T^+ \) is also given in the form

\[ T^+(r, t) = \Theta^+(t) - \frac{dT_{ad}}{dp} \Delta p \left( \frac{r}{R_{CMB}} \right)^2, \]  

(A4)

if the heat flow in the stratified layer is equal to the heat conducting down the adiabat. Here \( \Theta^+ \) is the potential temperature of the upper layer. By substituting the expression (A4) into the thermal conduction equation, we obtain

\[ \Theta^+(t) = T_0 - 6 \frac{dT_{ad}}{dp} \Delta p \left( \frac{t}{\tau_{therm}} \right), \]

(A5)

where \( T_0 \) is the temperature at the center of the Earth when the inner core began to form, and \( \tau_{therm} \) is the thermal diffusion time given by

\[ \tau_{therm} = \frac{R_{CMB}^2}{\kappa}, \]  

(A6)

where \( \kappa \) is the thermal diffusivity.

We can obtain the expression for the potential temperature of the convecting layer \( \Theta^-(t) \) by using the equilibrium condition at the ICB,

\[ T^-(r_{IC}, t) = T_L(r_{IC}, t), \]  

(A7)

Where \( T_L \) is the liquidus curve given by

\[ T_L(r_{IC}, t) = T_0 + \frac{\partial T_L}{\partial p} (p_{ICB} - p_0) + \frac{\partial T_L}{\partial C} (C_{ICB} - C_0). \]  

(A8)

From Eqs. (A1), (A3), (A7), and (A8), we obtain

\[ \Theta^-(t) = T_0 + \left( \frac{dT_{ad}}{dp} - \frac{\partial T_L}{\partial p} \right) \Delta p \left( \frac{R_{IC}}{R_{CMB}} \right)^2 \]

\[ + \frac{\partial T_L}{\partial C} \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2. \]  

(A9)

On the other hand, from Eqs.(11) and (12), we obtain the concentration of light elements in the convecting layer as

\[ C^- - C_0 = \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2. \]  

(A10)

Substituting Eq. (A10) into Eq.(A9), we have

\[ \Theta^-(t) = T_0 + \left( \frac{dT_{ad}}{dp} - \frac{\partial T_L}{\partial p} \right) \Delta p \left( \frac{R_{IC}}{R_{CMB}} \right)^2 \]

\[ + \frac{\partial T_L}{\partial C} \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2. \]  

(A11)

At the early stage of the inner core growth \( (R_{IC} \ll R_{CMB}) \), we can approximate Eq. (A11) as

\[ \Theta^-(t) = T_0 + \frac{\partial T_L}{\partial C} \Delta C \left( \frac{R_s}{R_{CMB}} \right)^2. \]  

(A12)
Finally, the time evolution of $R_s$ can be obtained from the heat budget of the convective layer, which is written as

$$
\frac{4\pi}{3} R_s^2 \rho C_p \frac{d\Theta}{dt} = \frac{4\pi}{3} \left( R_{IC}^2 \frac{dR_{IC}}{dt} \rho L + R_s^2 \rho C_p \frac{dT^+}{dr} \right) (R_s) + \frac{2}{3} \frac{dR_s}{dt} \rho C_p \left( T^+(R_s) - T^-(R_s) \right),
$$

(A13)

where $C_p$ is the specific heat and $L$ is the latent heat of the solidification of the inner core. Here we neglect the conduction from the inner core, and the release of the gravitational energy due to light element because it is small at the early stage of the inner core growth. In addition, we use $R_s \gg R_{IC}$, which is valid at the early stage. Using Eqs. (19), (A3), (A4), (A5), and (A12), we can rewrite Eq. (A13) as

$$
\left[ \frac{1}{3} \frac{\Delta C}{C_0} \left( \frac{5}{2} C - L \right) \left( \frac{R_s}{R_{CMB}} \right)^2 \left( \frac{5}{2} C - L \right) \left( \frac{R_s}{R_{CMB}} \right)^2 \right] + 3 \frac{t}{\tau_{therm}} \frac{1}{R_s} \frac{dR_s}{dt} = - \frac{1}{\tau_{therm}},
$$

(A14)

where $C$ and $L$ are non-dimensional parameters defined as

$$
C = \frac{\partial T_L}{\partial \Theta} \frac{C_0}{\frac{dT_{ad}}{dp} \Delta p},
$$

(A15)

and

$$
L = \frac{L}{\frac{dT_{ad}}{dp} \Delta p}.
$$

(A16)

The solution of Eq. (A14) is

$$
\frac{R_s}{R_{CMB}} = \sqrt{15 \frac{C_0 \Delta C}{\frac{5}{2} C} \left( \frac{5}{2} C - L \right)^{-1} \frac{t}{\tau_{therm}}},
$$

(A17)

which is Eq. (20).

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